Light elements as diagnostics on the structure and evolution of low- and intermediate-mass stars

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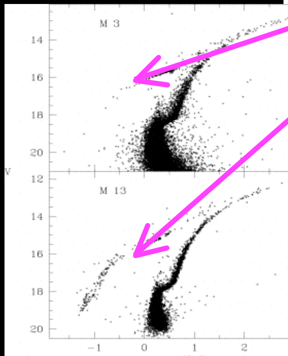
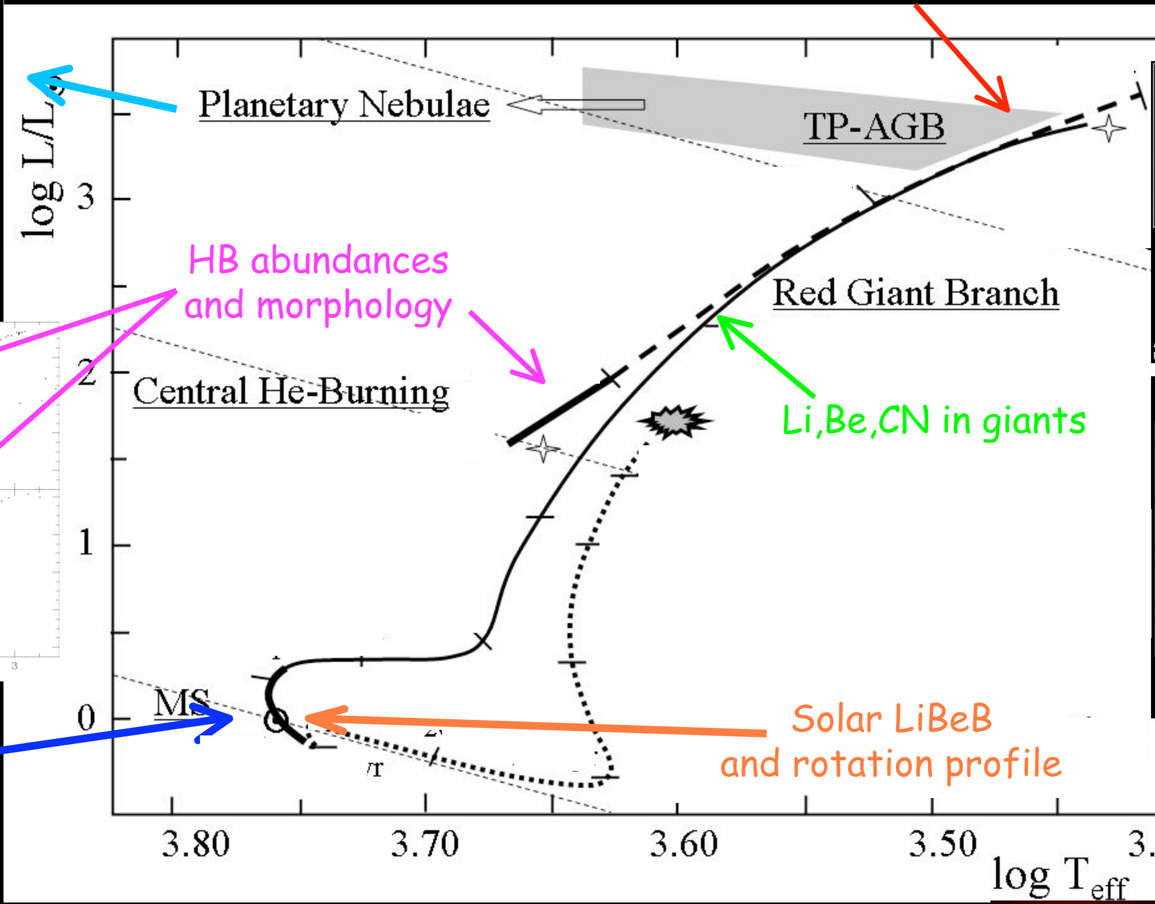
- Observational diagnostics
- Transport processes for angular momentum and chemicals
  - Atomic diffusion
  - Rotation-induced mixing
  - Internal gravity waves
- Towards a coherent picture of mixing over the HRD

Observational diagnostics

Abundances in PN ($^{12}C/^{13}C$, ^{3}He, ...)

White dwarf spins





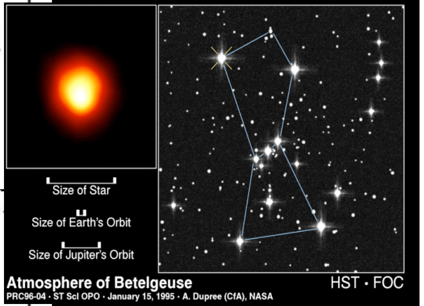
Li, F, primary ^{14}N O isotopes

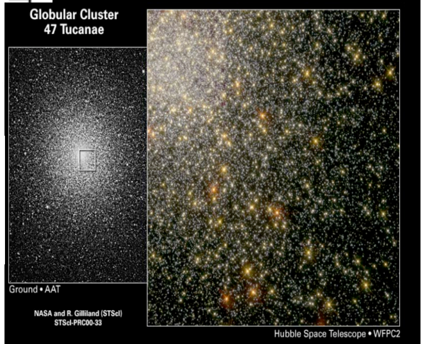
s-elements in TP-AGB

HB abundances and morphology

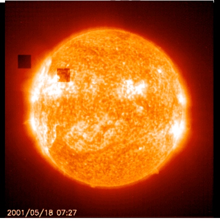
Li,Be,CN in giants

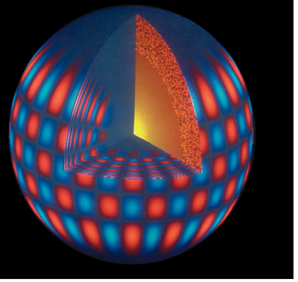
Solar LiBeB and rotation profile



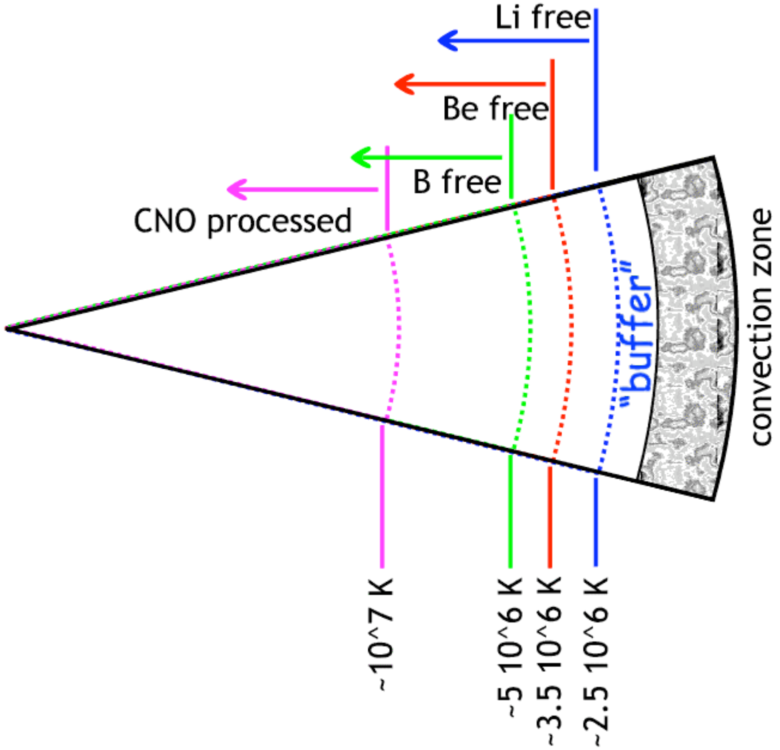


LiBeB in open clusters - Li plateau

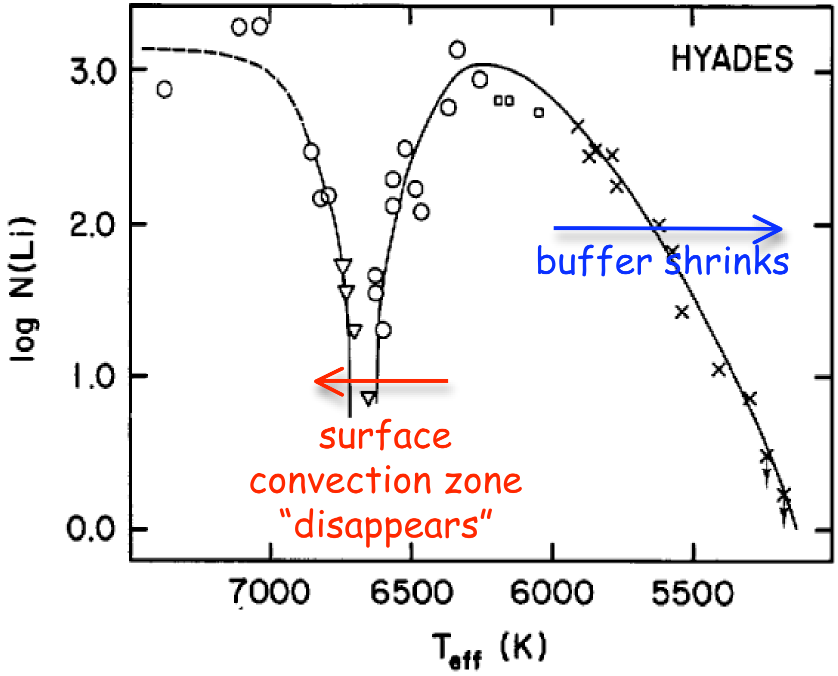




Abundance tomography and the Li Dip



The Lithium Dip

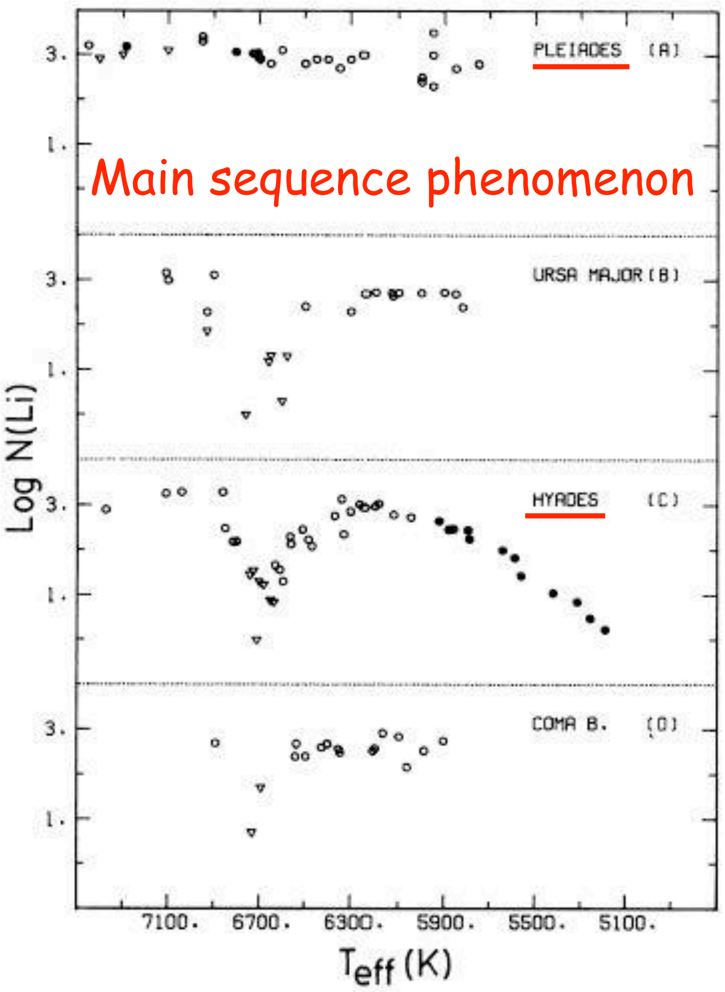


Boesgaard & Tripicco (1986)

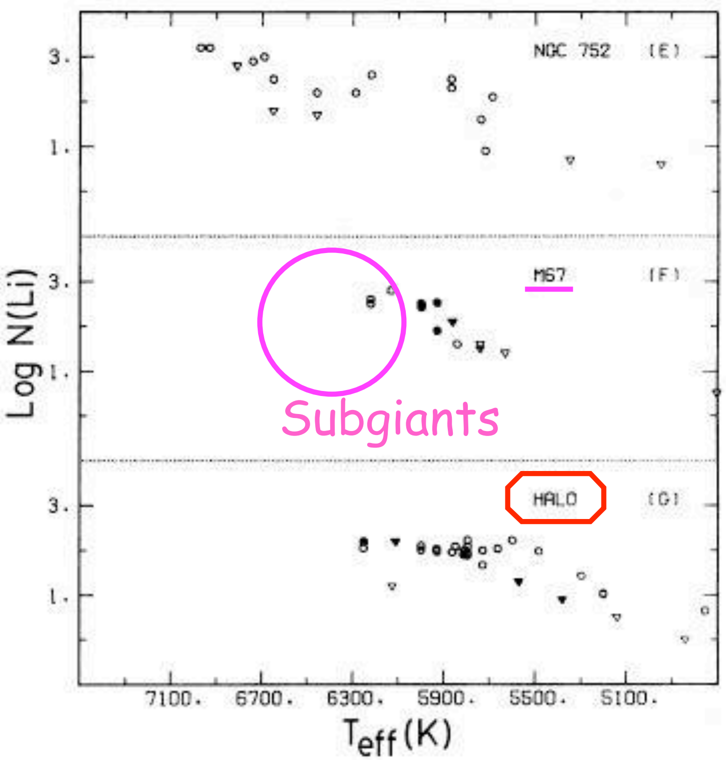
(first observed by Wallerstein, Herbig & Conti 1965)

The Li Dip

Observed in
all open clusters older than ~ 200 Myr
and in field stars (Balachandran 1995)



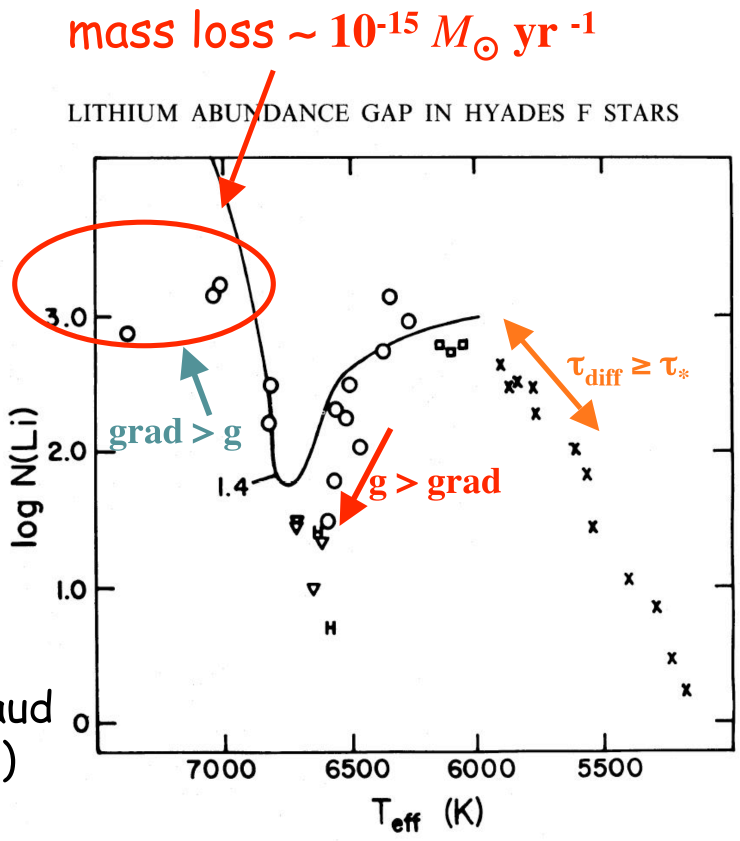
Main sequence phenomenon



Subgiants

Charbonneau & Michaud (1988)

The Li Dip – atomic diffusion



Michaud (1986)

Diffusion becomes increasingly efficient with decreasing density below the CE, i.e., with increasing T_{eff}

Problem:
Heavy elements are also expected to settle down → Not observed (here e.g. for C and Na)

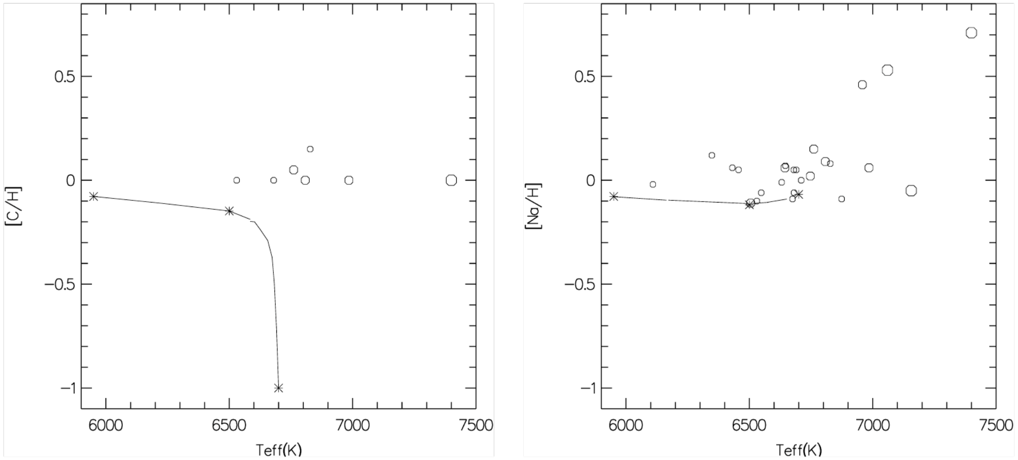
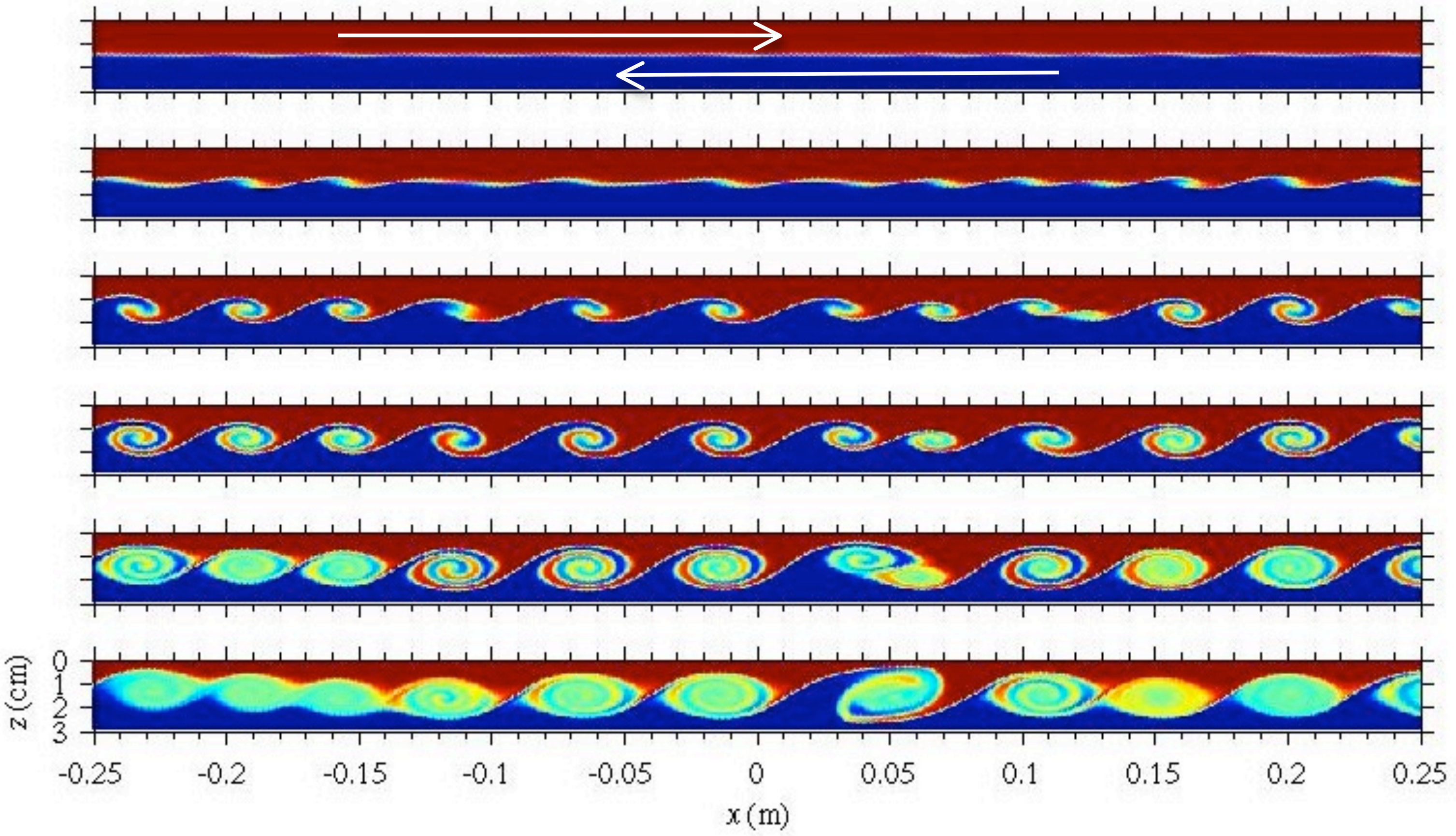


Fig. and data in the Hyades from Varenne & Monier (99)
Predictions by Turcotte et al. (98)

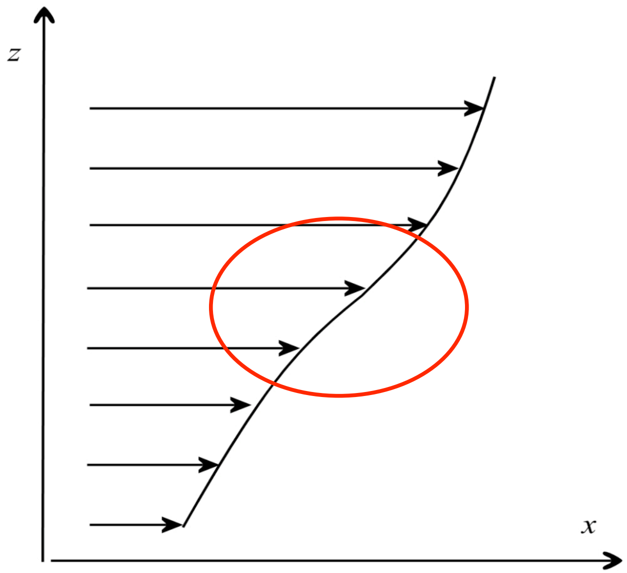
Rotation-induced mixing
The shear instability



(Dan Kelly)

Rotation-induced mixing

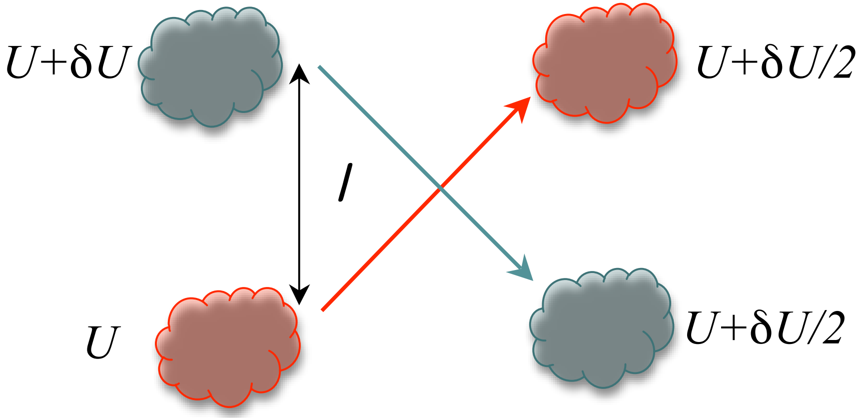
The shear instability



Rayleigh instability criterion:

$$\frac{\mathrm{d}^2 u}{\mathrm{d}z^2} = 0$$ Rayleigh (1881)

⇒ condition for linear instability



Kinetic energy: $\frac{1}{4}\, m\, \delta U^2$

Work against gravity: $g\, m\, l$

Richardson stability criterion:

(dynamical stability)

$$Ri = \frac{N^2}{(\mathrm{d}u/\mathrm{d}z)^2} > Ri_{\mathrm{crit}} = \frac{1}{4}$$

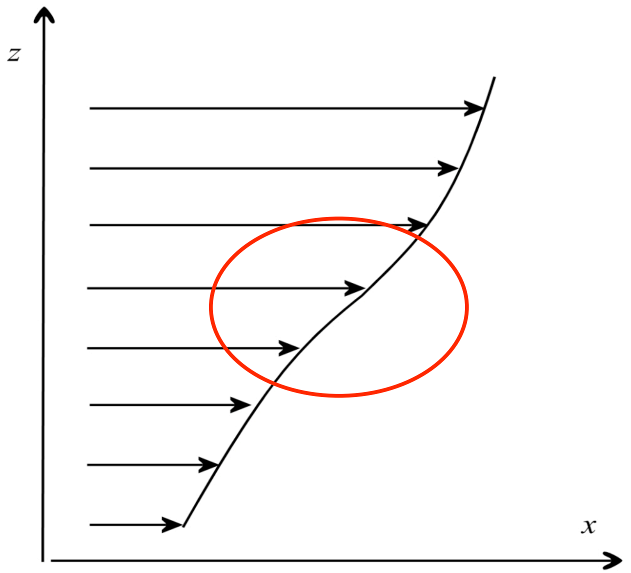
Will occur:

⇨ along equipotentials ($D_h \gg D_v$)

⇨ for very large shears

Rotation-induced mixing

The shear instability



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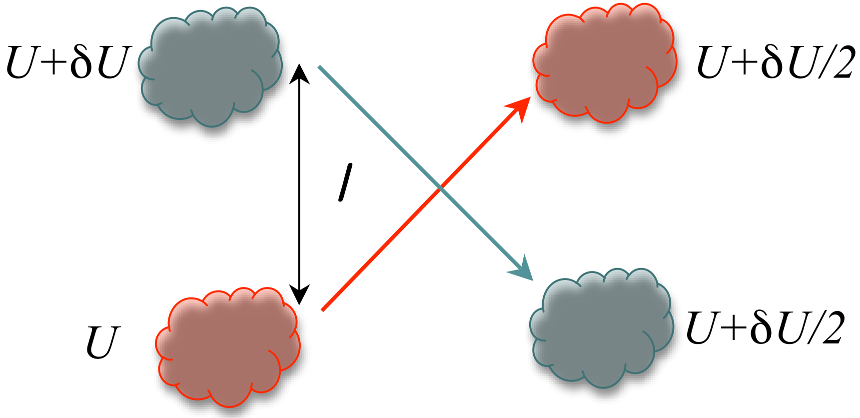

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Richardson stability criterion:

(dynamical stability)

$$Ri = \frac{N^2}{(\mathrm{d}u/\mathrm{d}z)^2} > Ri_{\mathrm{crit}} = \frac{1}{4}$$

Non-adiabatic Richardson criterion:

$$\frac{\nu l}{K_T} N_T^2 + \frac{\nu l}{K_\mu} N_\mu^2 < Ri_{\mathrm{crit}} \left(\frac{\mathrm{d}u}{\mathrm{d}z} \right)^2$$

Maeder 1995; Talon, Zahn 1997

Rotation-induced mixing
Meridional circulation

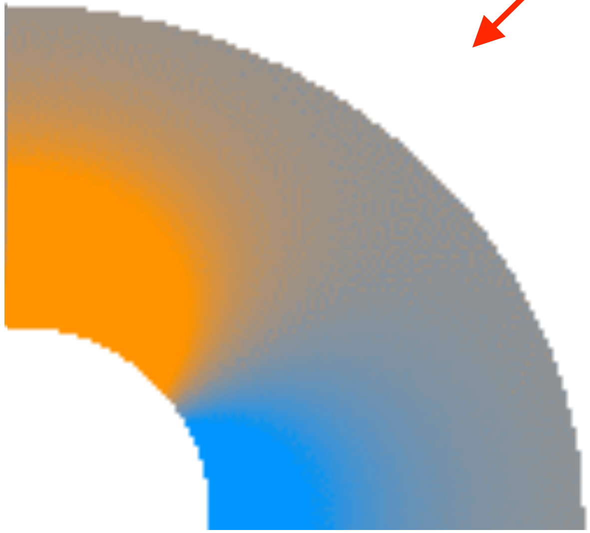
Thermal equilibrium in a rotating star:

$$0 = \nabla\left(\chi\nabla T\right) + \rho\varepsilon$$

von Zeipel
(1924)

heat flux
divergence

energy
generation



This source can be compensated for by a large scale advection of entropy

Eddington-Sweet meridional circulation

$$\boxed{\rho T\vec{u}\cdot\nabla s} = \nabla\left(\chi\nabla T\right) + \rho\varepsilon$$

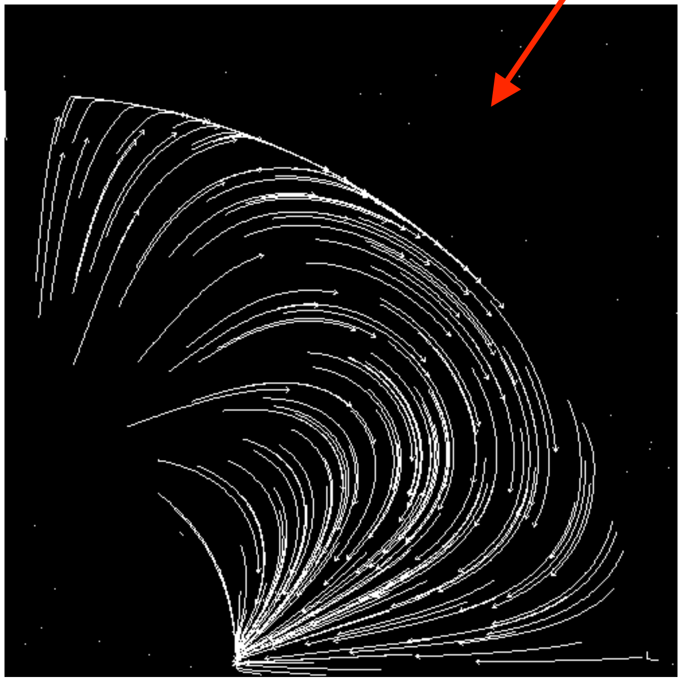
Vogt (1925),
Eddington (1925)

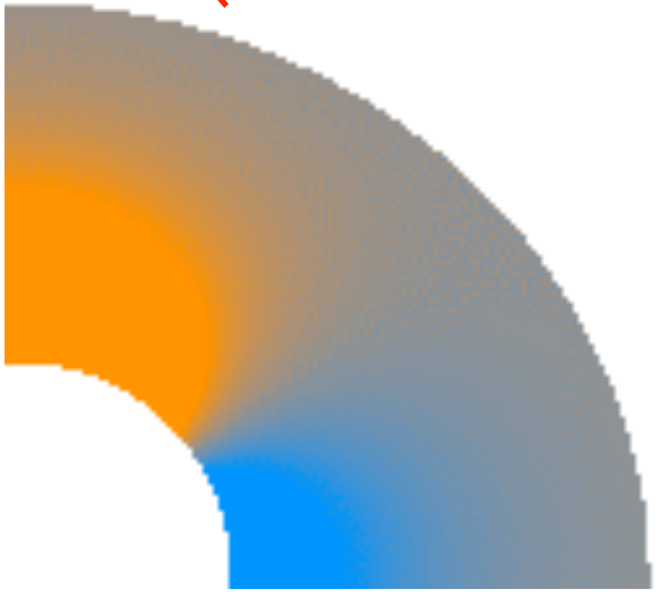
Rotation-induced mixing
Meridional circulation

In solid body rotation:

$$\rho T\vec{u}\cdot\nabla s = \nabla\left(\chi\nabla T\right)$$

Sweet (1950)





Other theoretical studies:

Öpik 1951; Mestel 1953, 1966; Roxburgh 1966; Osaki 1972; Sakurai 1975; Busse 1981; Tassoul & Tassoul 1982

Rotation-induced mixing
Meridional circulation

⇨ Must treat the advection of angular momentum by MC
assume $D_h \gg D_v$ to transform the 2D problem into a 1D problem

Evolution of angular momentum:

$$\rho \frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega\right) = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho r^4 \Omega u_r\right] + \frac{1}{r^4}\frac{\partial}{\partial r}\left[\rho \nu_v r^4 \frac{\partial \Omega}{\partial r}\right]$$

circulation turbulence

Evolution of chemical species:

$$\rho \frac{\mathrm{d}c}{\mathrm{d}t} = \rho\, c_{\mathrm{nuc}} + \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \rho V_{ip} c\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho r^2 \left(D_{\mathrm{eff}} + D_v\right)\frac{\partial c}{\partial r}\right]$$

nuclear transformation atomic diffusion

Zahn 1992; Maeder, Zahn 1998; Mathis, Zahn 2004

- There exists a stationnary solution (core rotates 1.4 times faster than the surface)
- If you extract angular momentum from the surafce, as in the Sun, you create internal shear that should increase circulation and turbulent mixing

Rotation-induced mixing

Shear and baroclinic instabilities and meridional circulation as a diffusive process:

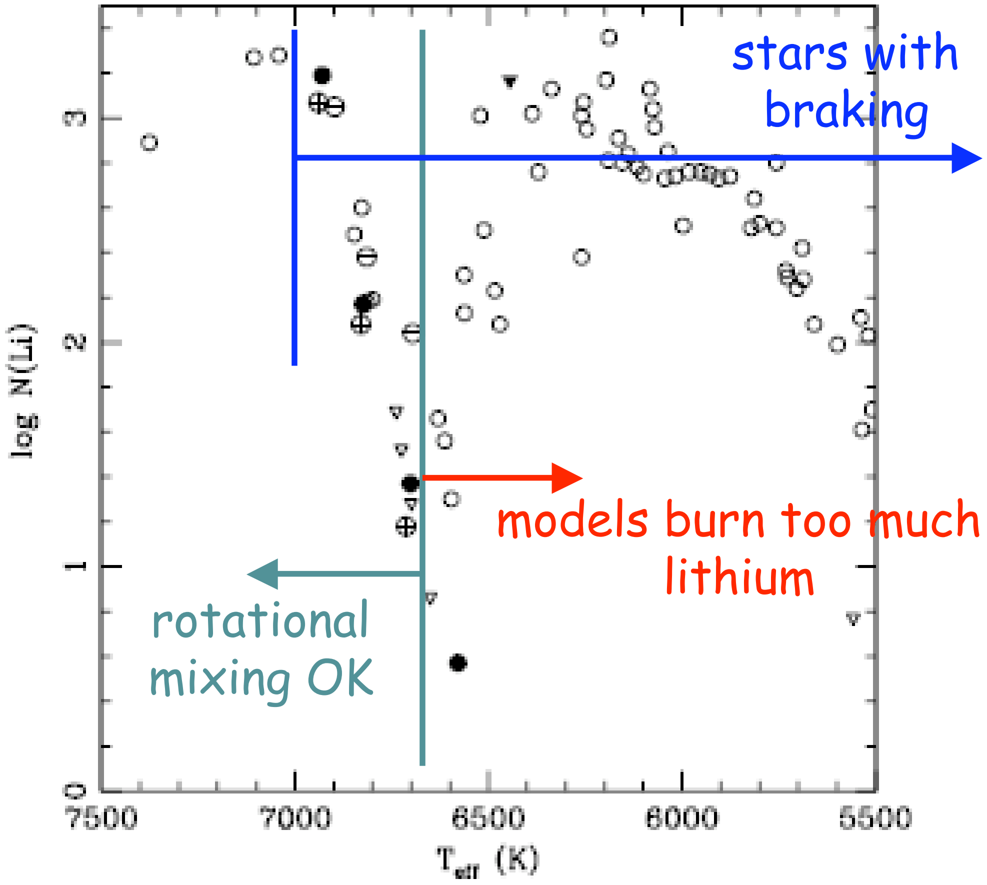
Endal, Sofia 1976, 1978, 1979; Langer 1991; Fliegner, Langer 1995, Heger, Langer, Woosley 2000; Yoon, Langer, Scheithauer 2004 (accreting white dwarfs), Siess, Goriely, Langer 2004 (nucleosynthesis in AGB stars)

Shear and meridional circulation as an advective process:

Talon, Zahn, Maeder, Meynet 1997; Meynet, Maeder 2000, 2003; Maeder, Meynet 2000, 2001; Vasquez, Leitherer, Schaerer, Meynet, Maeder 2007

Largely successful in explaining massive stars abundance anomalies

Rotation-induced mixing in low-mass stars



stars with braking

models burn too much lithium

rotational mixing OK

Talon & Charbonnel (1998)

Boesgaard (1987):

connection between Li Dip and surface braking

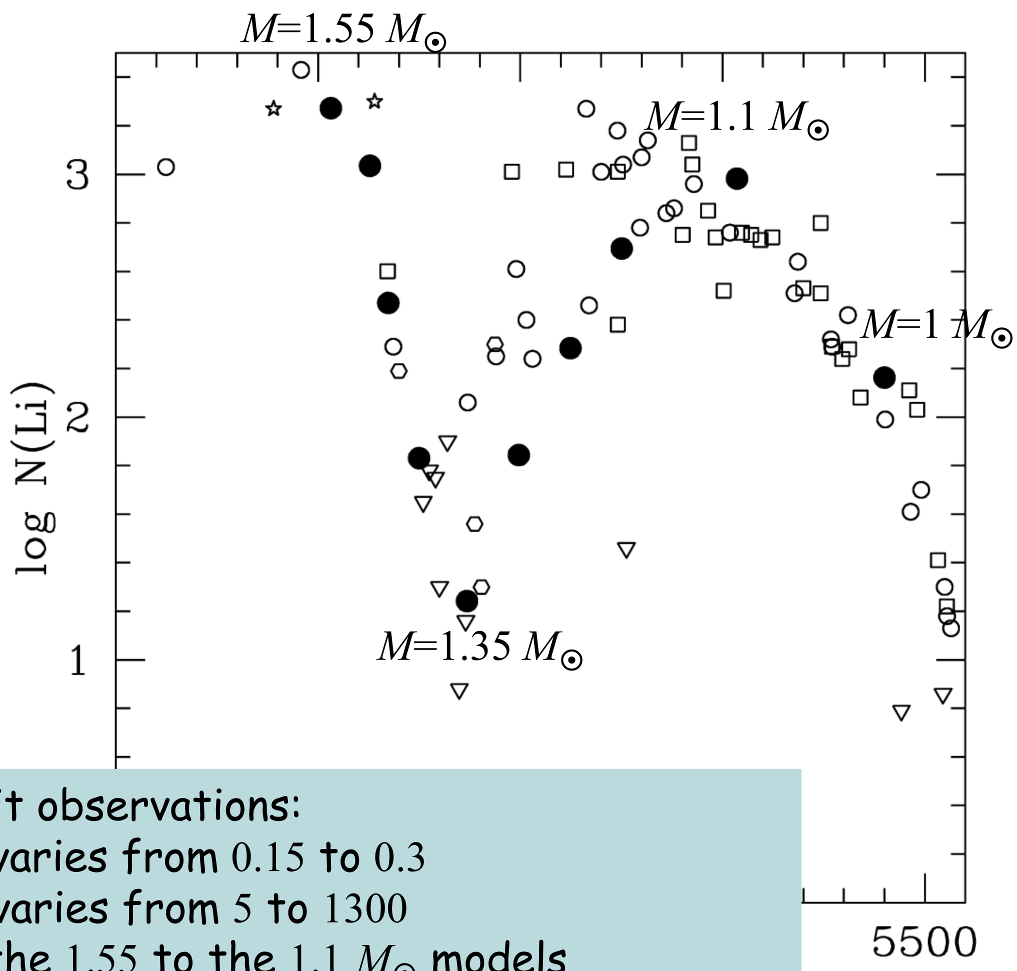
Shear and meridional circulation as an advective process:

Talon & Charbonnel 1998;
Palacios, Talon, Charbonnel, Forestini 2003

Also mixing in giants:

Palacios, Charbonnel, Talon, Siess 2006

Rotation-induced mixing in low-mass stars



To fit observations:
- C_v varies from 0.15 to 0.3
- C_h varies from 5 to 1300

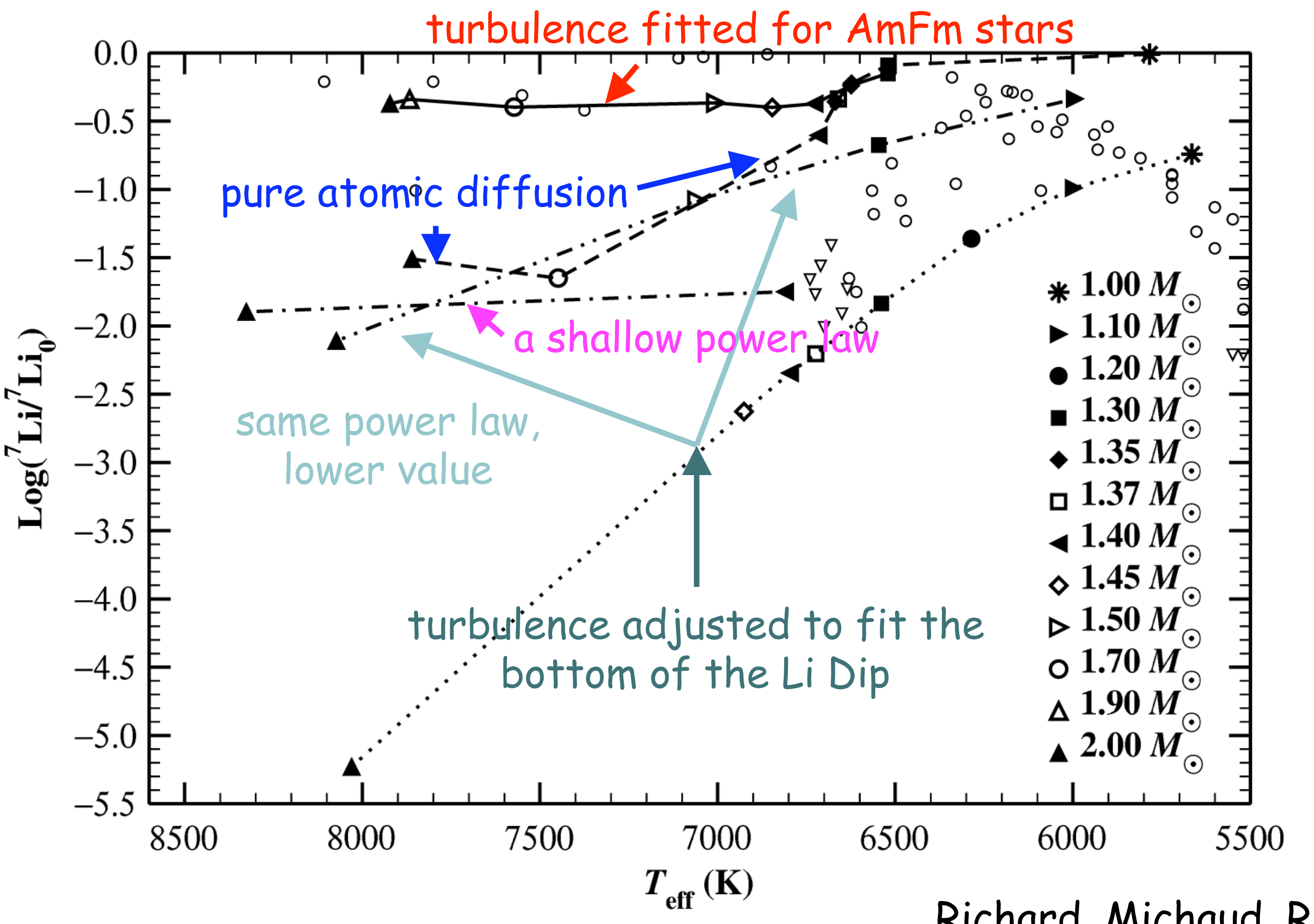
for the 1.55 to the $1.1\ M_\odot$ models
Variation is even larger for the 1 $M_\odot$ model.

Théado & Vauclair (2003)

Models (filled points) with:

- Meridonal circulation assuming solid body rotation
- Helium settling assumed to slow down meridonal circulation
- Turbulent mixing proportionnal to U_r and inversely proportioanl to two adjustable parameters C_v and C_h

Rotation-induced mixing in low-mass stars



turbulence fitted for AmFm stars

pure atomic diffusion

a shallow power law

same power law, lower value

turbulence adjusted to fit the bottom of the Li Dip

Models with:

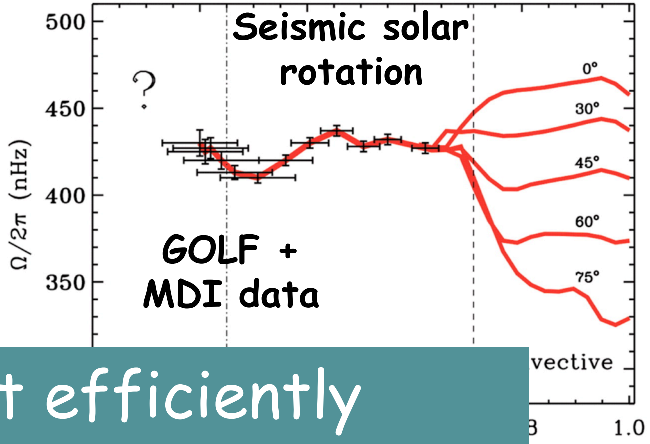
- Atomic diffusion (settling + radiative acceleration)
- ad-hoc turbulence (as a power law)

various lines correspond to various turbulence coefficients

Richard, Michaud, Richer & Talon (unpublished)

Rotation-induced mixing in the Sun

García et al. (2007)



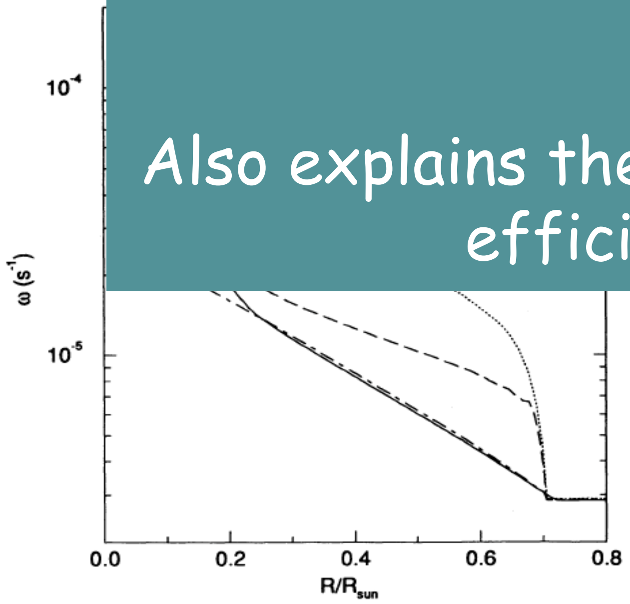
Shear and baroclinic instabilities and meridional circulation as a diffusive process:

Endal, Sofi
1989; Chabo

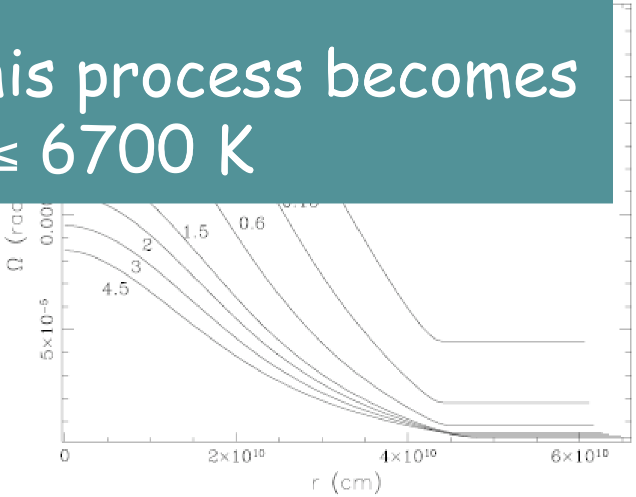
as an

Need physical process that efficiently transports angular momentum but not chemicals

Also explains the Li dip is this process becomes efficient at $T_{eff} \le 6700$ K



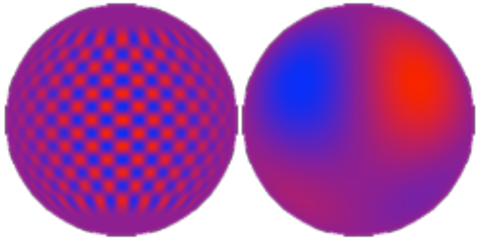
Chaboyer, Demarque & Pinsonneault (1995)



Talon (1997)

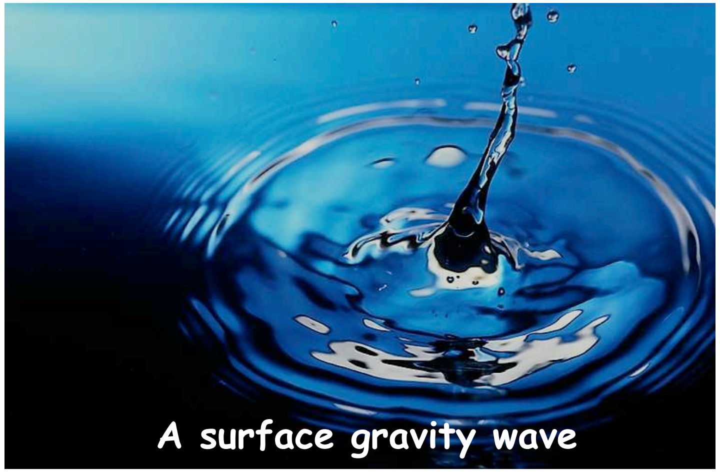
Internal gravity waves

g-mode
pulsation

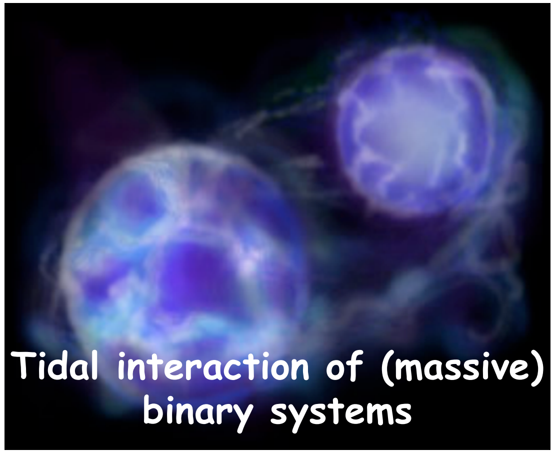


Earth's atmosphere

Bretherton
(1969)



A surface gravity wave



Tidal interaction of (massive) binary systems

Tidal interaction of (massive) binary systems - Zahn (1970, 1975, 1976), Goldreich & Nicholson (1989)

Mixing of chemicals - Press (1981) , Garcia-Lopez & Spruit (1991), Schatzman (1993), Montalban (1994), Montalban & Schatzman (1996, 2000), Young et al. (2003), Young & Arnett (2005)

Momentum redistribution by standing waves in Be stars - Ando (1986), Lee (2006)

Momentum redistribution in the Sun by travelling IGWs - Schatzman (1993), Kumar & Quataert (1997), Zahn, Talon & Matias (1997), Kumar, Talon & Zahn (1999), Talon, Kumar & Zahn (2002), Denissenkov et al. (2008)

Momentum redistribution and interaction with meridional circulation and shear turbulence - Talon & Charbonnel (2003, 2004, 2005), Charbonnel & Talon (2005)

Internal gravity waves
Properties

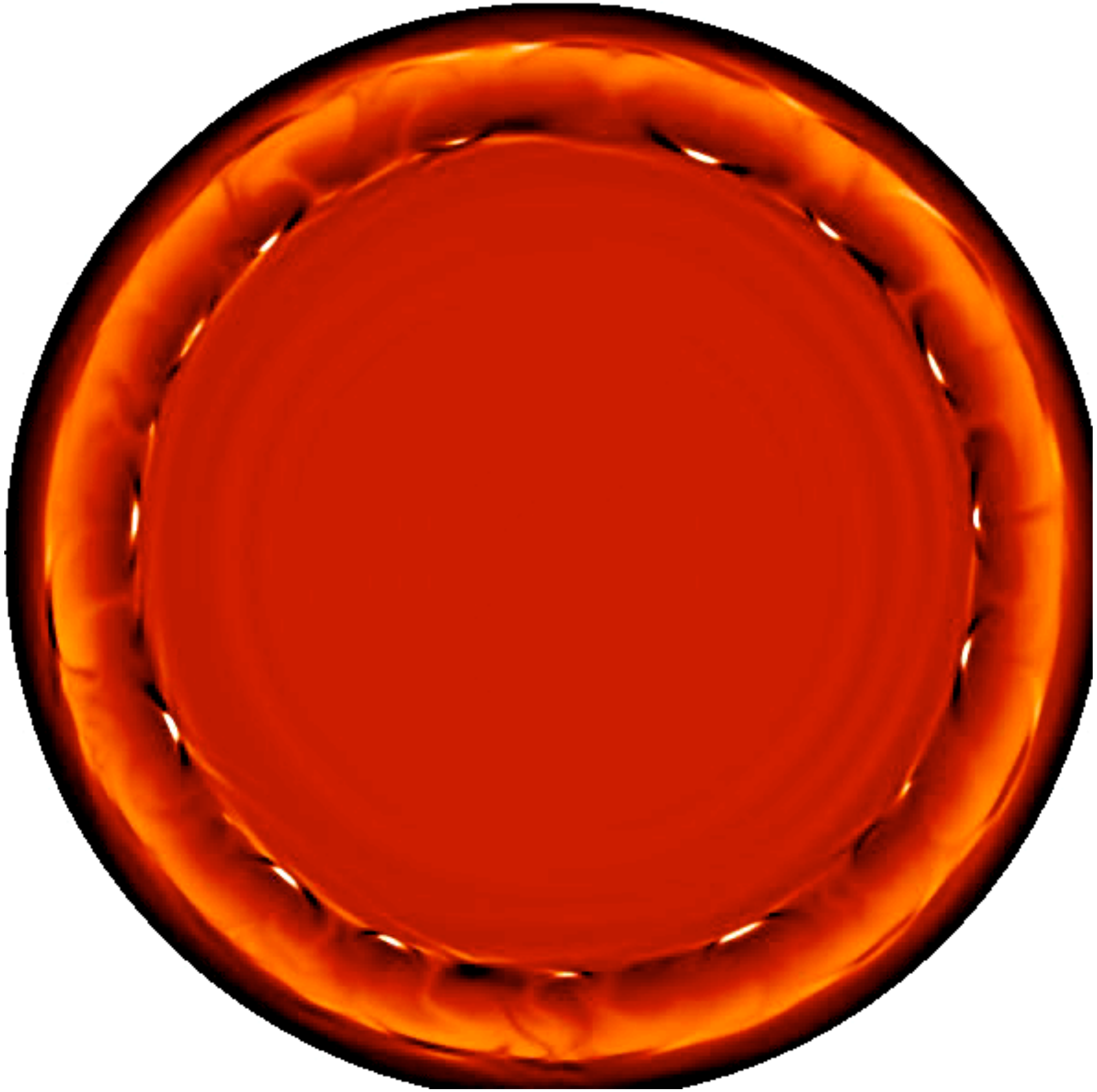
- Propagate in stratified media;
  - restoring force Þ buoyancy Þ vertical and horizontal differ
  - $0 < \omega < N$
  - $\vec{v}_g \perp \vec{k}$

Brunt-Väisälä frequency: natural oscillation frequency of a displaced element in a stratified region

- Excited by turbulence (e.g. close to convective zones);
- Conserve their momentum (or angular momentum) as long as they are not dissipated;
- They can transport angular momentum from the region where they are excited to where they are damped.

Internal gravity waves
Excitation

Rogers & Glatzmeier
(2005)



Other numerical simulations: Hurlburt, Toomre, Massaguer 1986; Hurlburt, Toomre, Massaguer, Zahn 1994; Andersen 1994; Nordlund, Stein, Brandenburg 1996; Kiraga, Rozyczka, Stepien, Jahn, Muthsam 2000; Dintrans, Brandenburg, Nordlund 2005

Theoretical work: based on solar p-mode models: Goldreich, Murray, Kumar 1994, Kumar, Quataert 1997, Belkacem et al. in prep.

Internal gravity waves
Excitation

Excitation by the turbulent interface

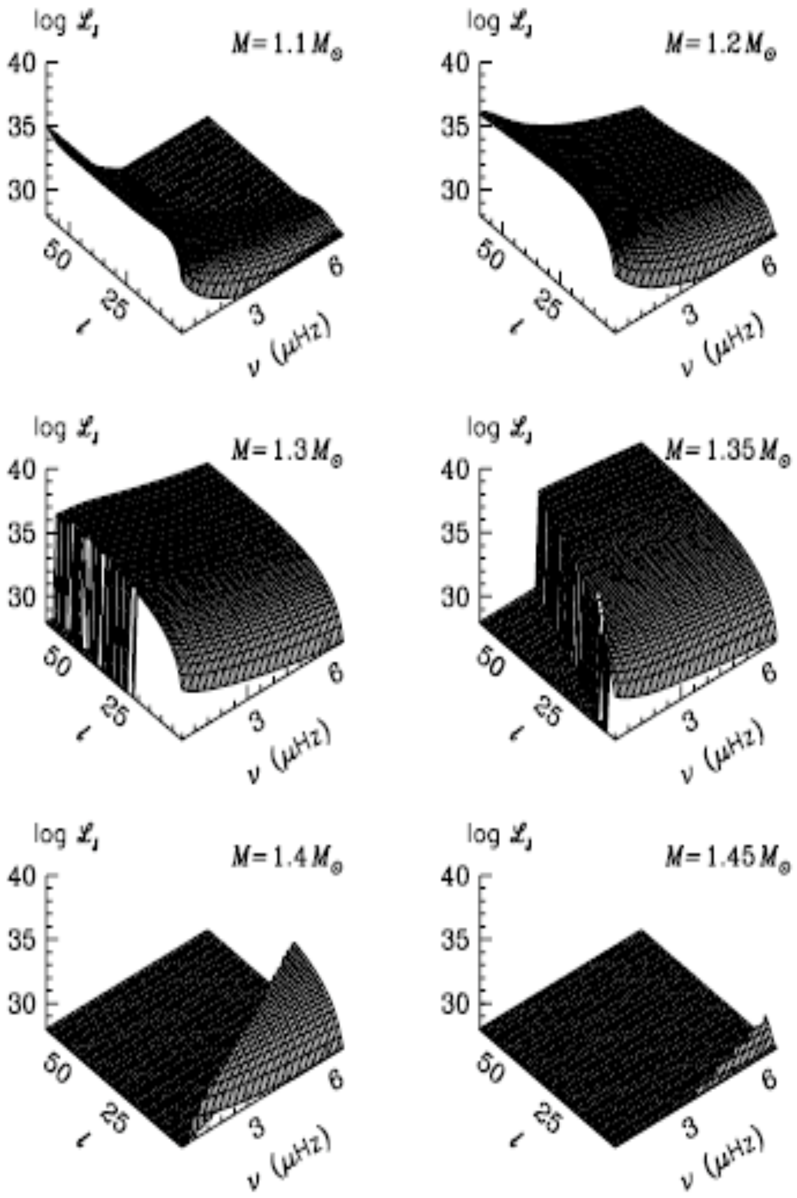
(modelled using the mixing length…)

Goldreich, Murray & Kumar (1994)

$$F_{J\ell,m} = \frac{2m}{\sigma} F_{K\ell,m}$$

Most of the momentum is carried by low-frequency waves

Significant momentum luminosity in low-order waves that penetrate deep into the interior



Talon & Charbonnel (2003)

Internal gravity waves
Angular momentum transport

- In stars, internal waves are mostly dissipated by thermal diffusivity

$$F_J(\sigma,\ell,m,r) = \boxed{F_J(\sigma,\ell,r_{zc})}\exp\left[-\tau(\sigma,\ell,m,r)\right]$$

flux at the base of the CZ

$$\tau(\sigma,\ell,m,r) = \left[\ell(\ell+1)\right]^{\frac{3}{2}}\int_{r}^{r_{zc}} K_T \frac{N N_T^2}{\sigma^4}\frac{\mathrm{d}r}{r^3}$$

thermal diffusivity

frequency in the rotating frame

$$\sigma(r) = \omega - m\Delta\Omega(r)$$

local frequency is Doppler shifted if there is differential rotation

Internal gravity waves
Angular momentum transport

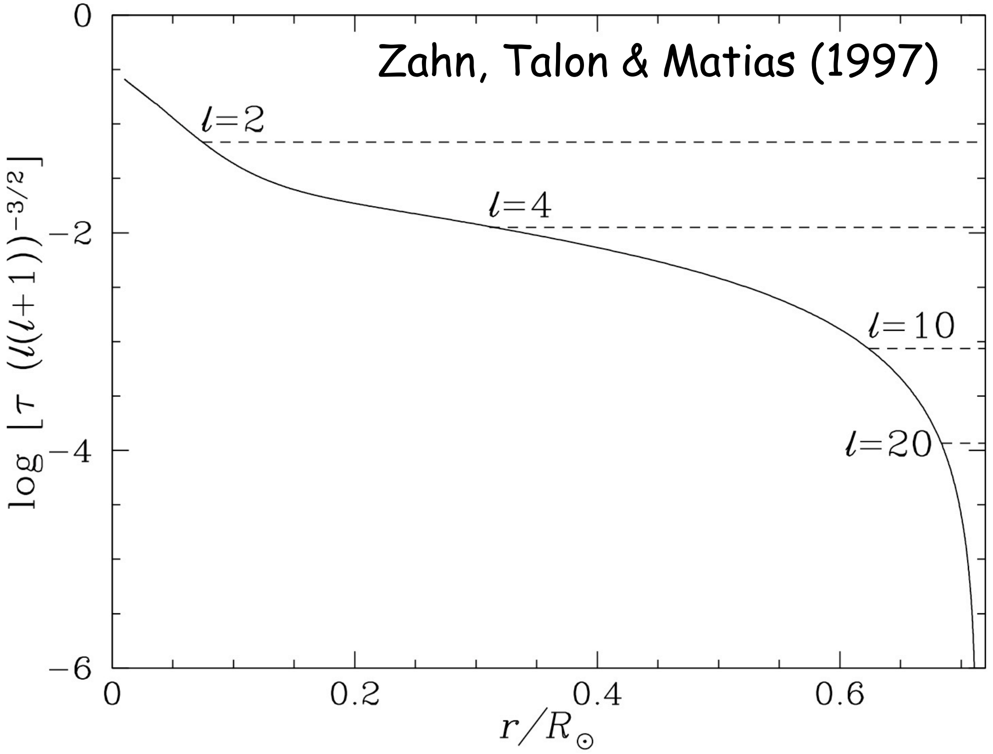
- In stars, internal waves are mostly dissipated by thermal diffusivity

$$\tau(\sigma,\ell,m,r) = \left[\ell(\ell+1)\right]^{\frac{3}{2}} \int_r^{r_{zc}} K_T \frac{N N_T^2}{\sigma^4} \frac{\mathrm{d}r}{r^3}$$

Evaluation of the **damping factor** τ for a frequency of 1 μHz in a solar model.

15 days

The depth corresponding to an attenuation by a factor 1/e is shown for various degrees ℓ



High ℓ waves are damped very close to the CZ

Internal gravity waves
Angular momentum transport

- if prograde ($m > 0$) and retrograde ($m < 0$) waves are equally excited and there is no differential rotation

  ➔ No net angular momentum deposition

- if there is differential rotation, $m > 0$ and $m < 0$ waves deposit their angular momentum at different locations

  ➔ Waves increase the local differential rotation

- high ℓ waves are damped very close to the CZ

Internal Gravity Waves
Short timescales in the Sun

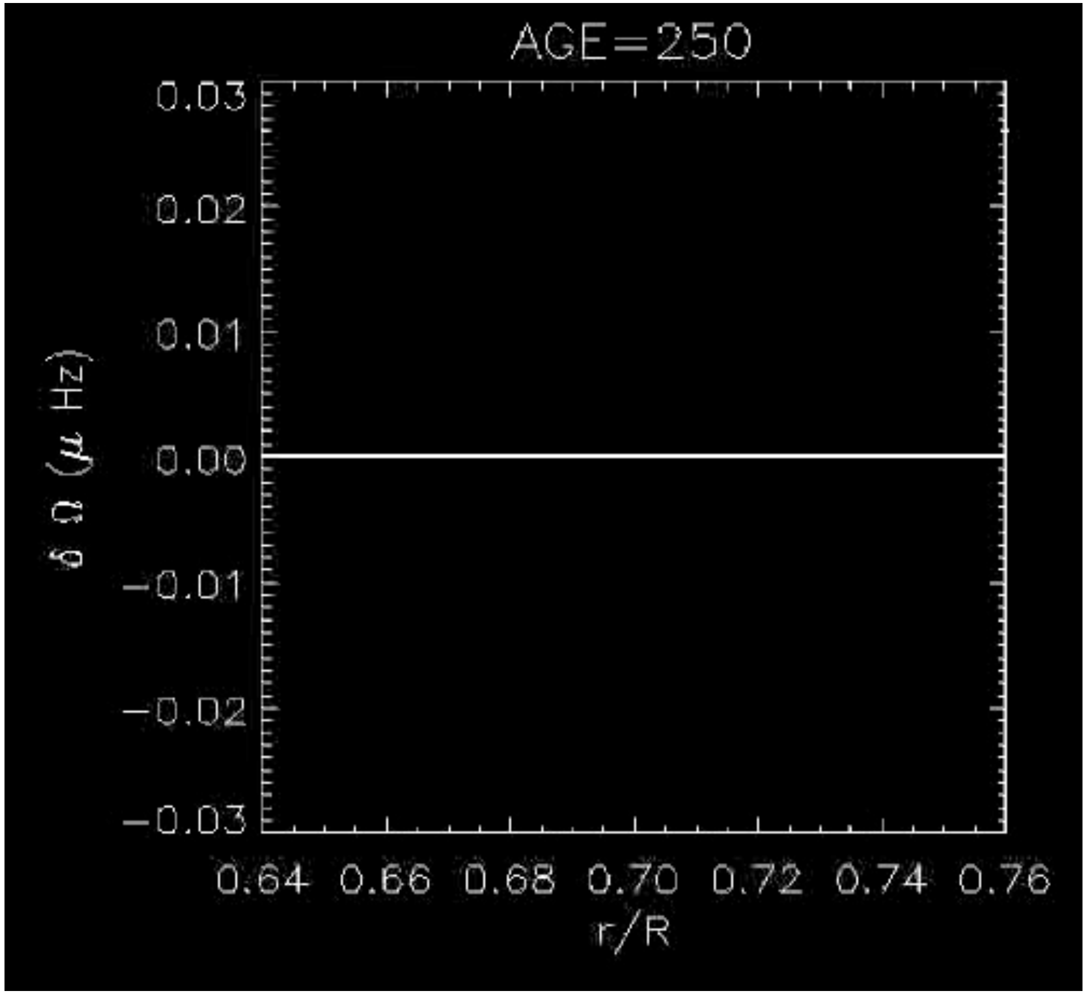
- on short time-scales

➔Shear layer oscillation (SLO) dominated by high-degree waves

➔For the Sun, τ ≈ few years

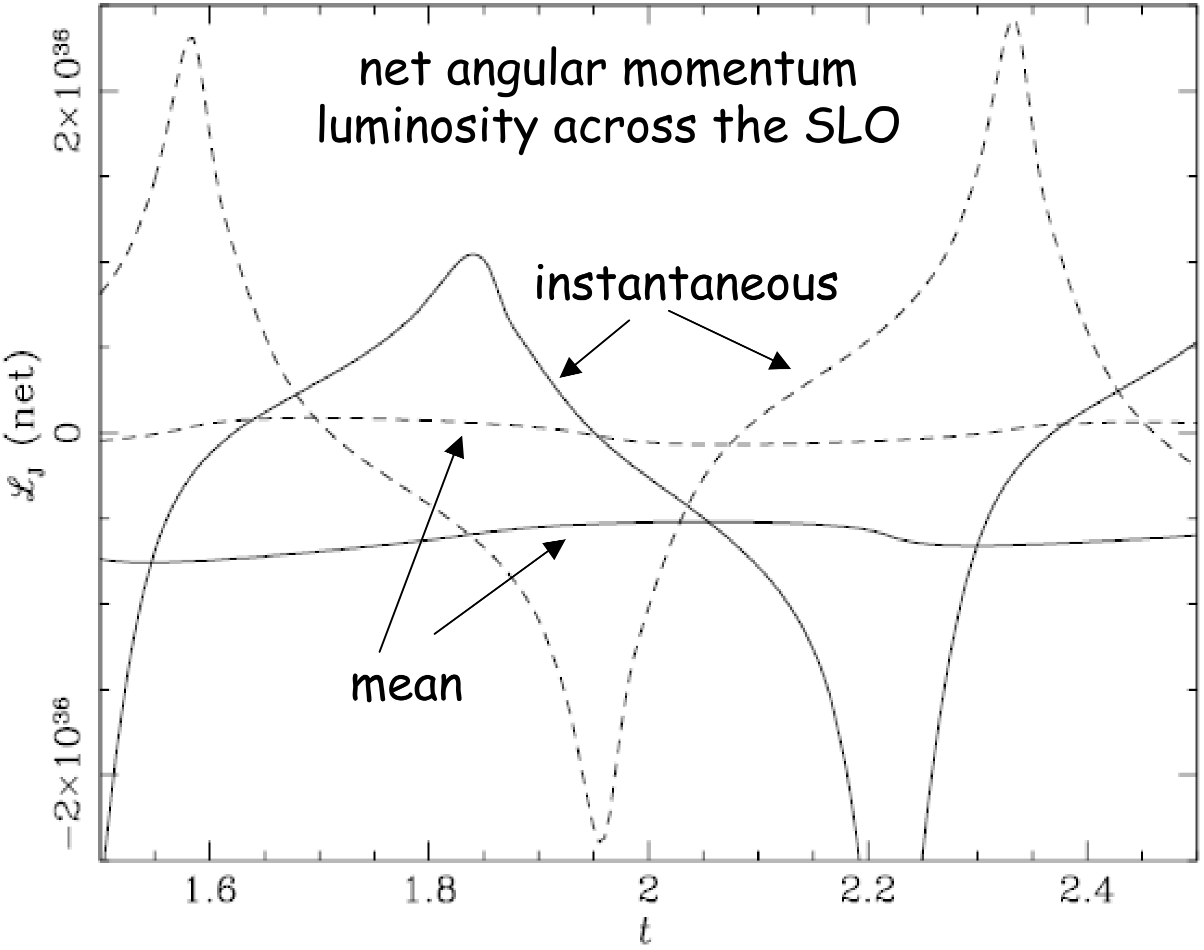
SLO discussed by:

Ringot 1998; Kumar, Talon, Zahn 1999; Kim, MacGregor 2001



Talon & Charbonnel (2005)

Internal Gravity Waves
Long timescales in the Sun



net angular momentum luminosity across the SLO

instantaneous

mean

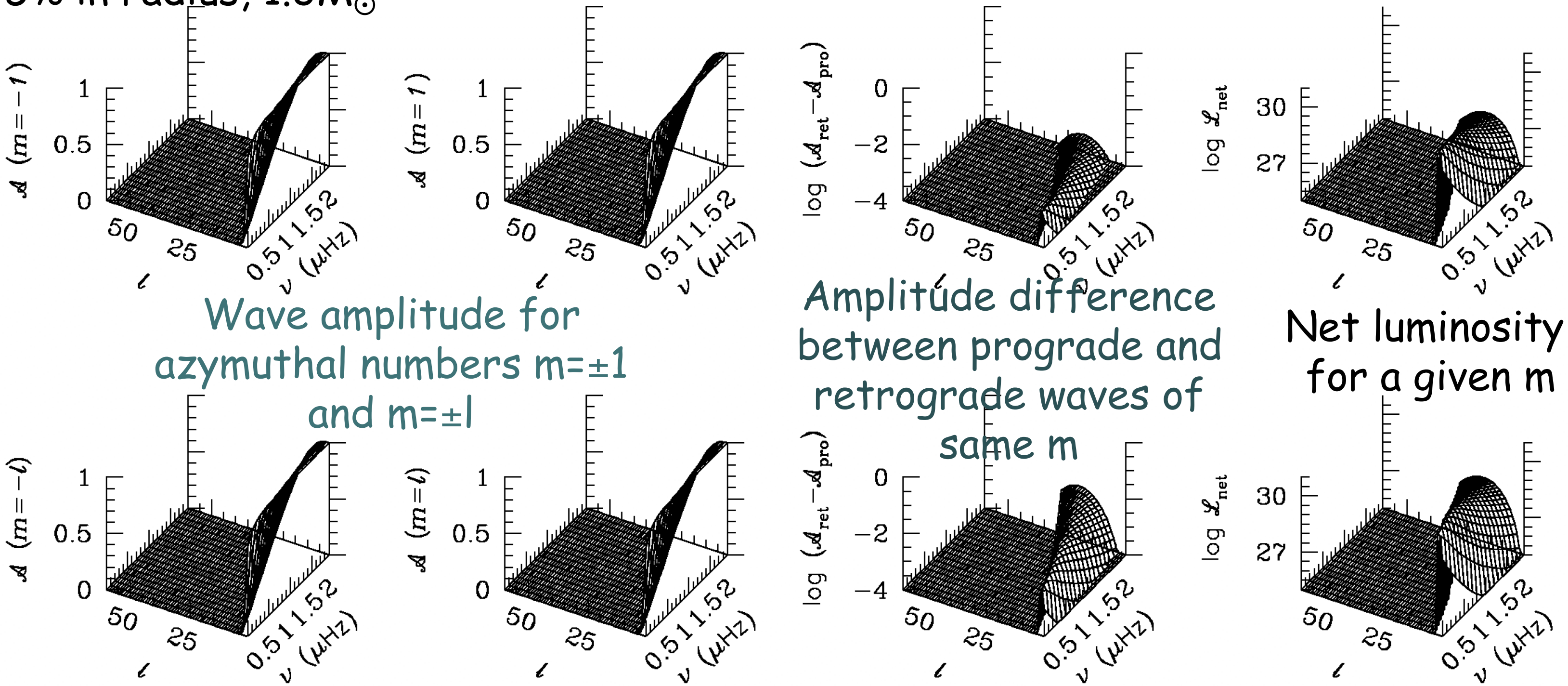
——— large differential rotation

- - - - - small differential rotation

Talon & Charbonnel (2003)

The SLO, by Doppler shifting to lower frequency both prograde and retrograde waves, acts as an efficient asymmetric IGW filter

Wave characteristics below the shear layer for a differential rotation of $\delta\Omega$=0.1μHz over 5% in radius, 1.3$M_\odot$



Wave amplitude for azymuthal numbers m=±1 and m=±l

Amplitude difference between prograde and retrograde waves of same m

Net luminosity for a given m

Low-degree, low-frequency waves, do conserve significant amplitude below the SLO. They penetrate into the deep interior and have the strongest impact on the secular redistribution of AM in the RZ

Talon & Charbonnel (2005)

Internal Gravity Waves
Long timescales in the Sun

- on long time-scales

➔Low-degree, low-frequency waves penetrate into the deep interior

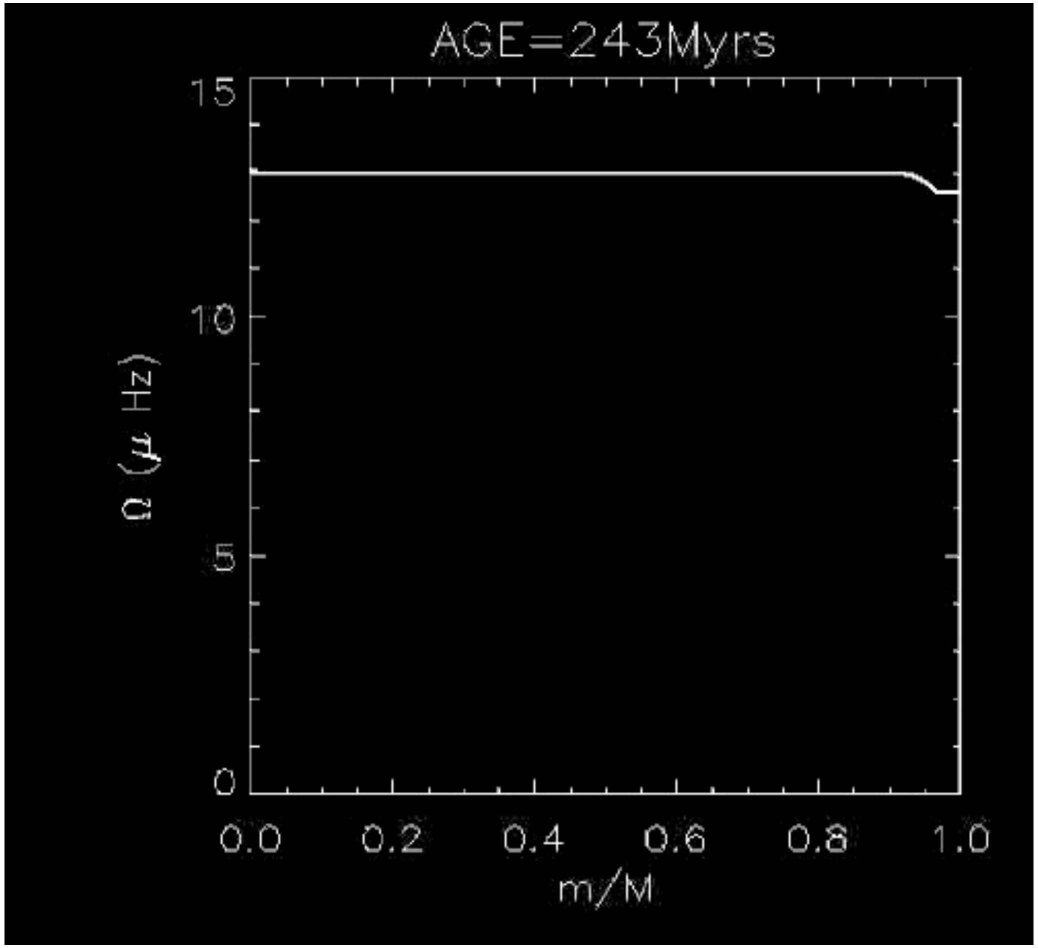
Momentum extraction first demonstrated by:

Talon, Kumar, Zahn 2002

Close to the core:

$j \rightarrow 0$ local oscillation?

Rogers & Glatzmaier 2006



Talon & Charbonnel (2005)

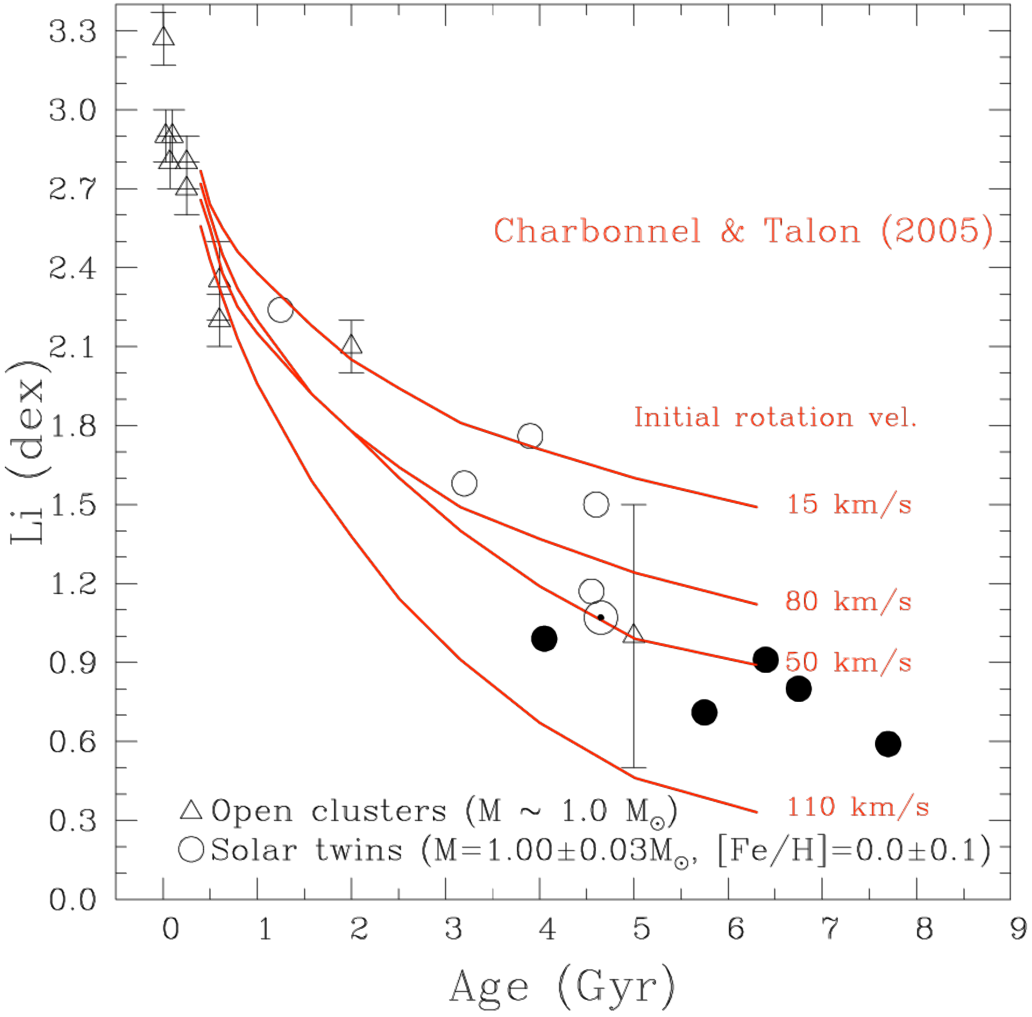
Internal Gravity Waves
Li in the Sun

Angular momentum transport is dominated by IGWs and meridional circulation

Lithium depletion is dominated by internal shears

Initial velocity depends on:
- initial angular momentum in the cloud
- disk coupling

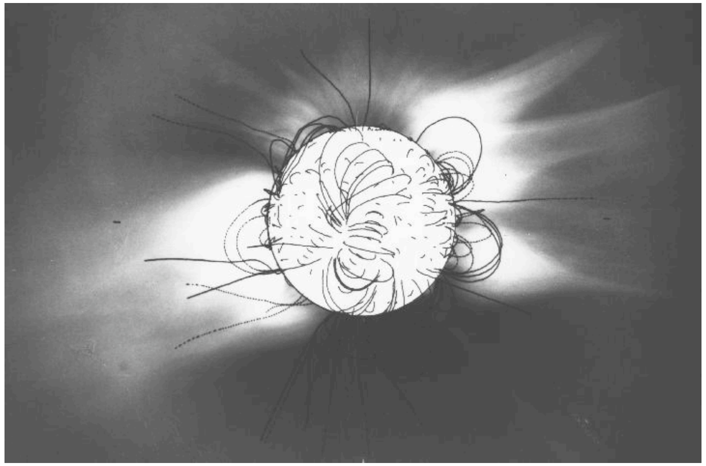
Meléndez et al. (2009)
See poster at this conf.



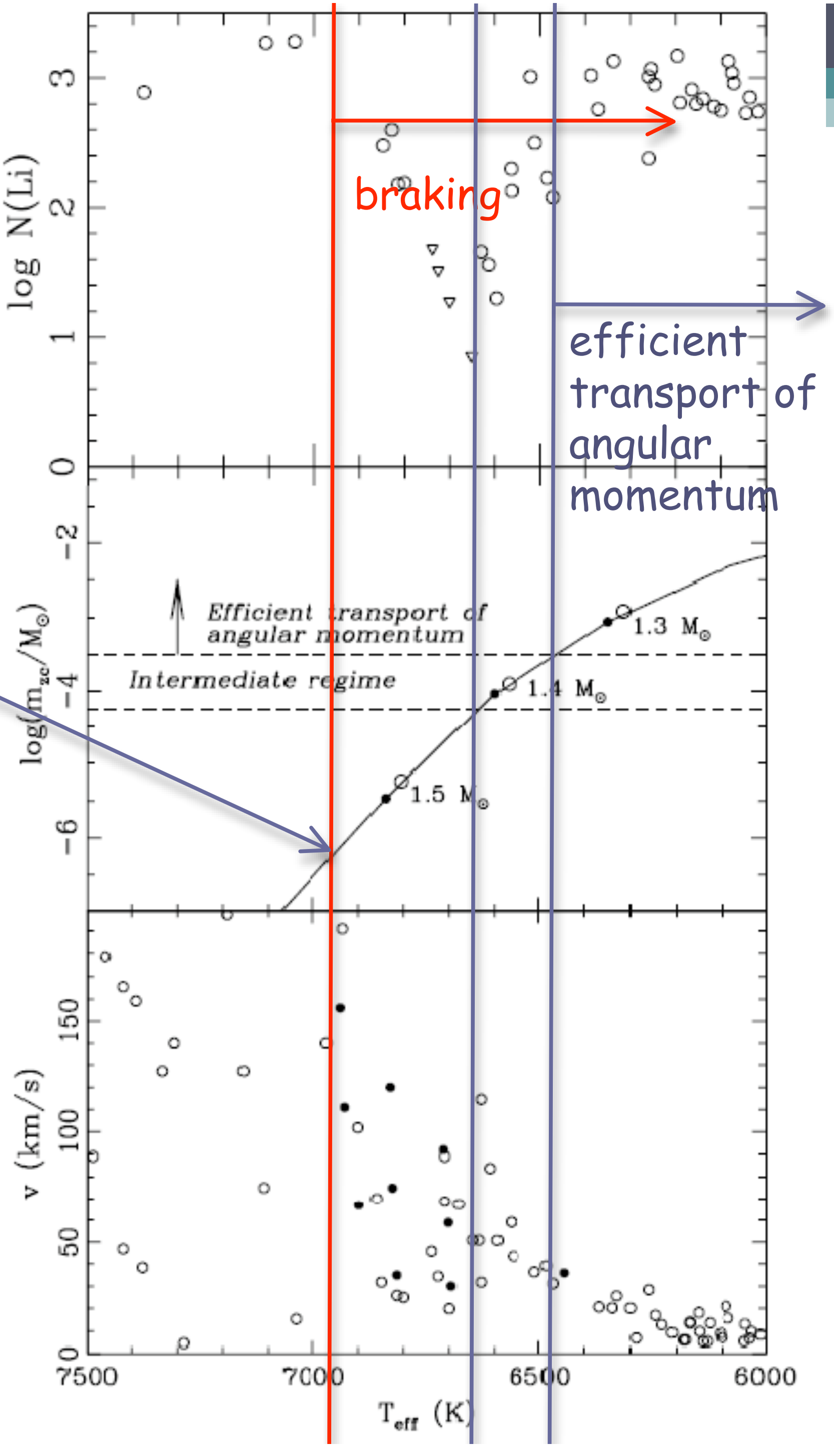
The Li Dip

A clue for the apparition of a new transport process for angular momentum

A deep enough surface
convective region is required to
sustain a dynamo and produce a
surface magnetic field that is
then responsible for braking

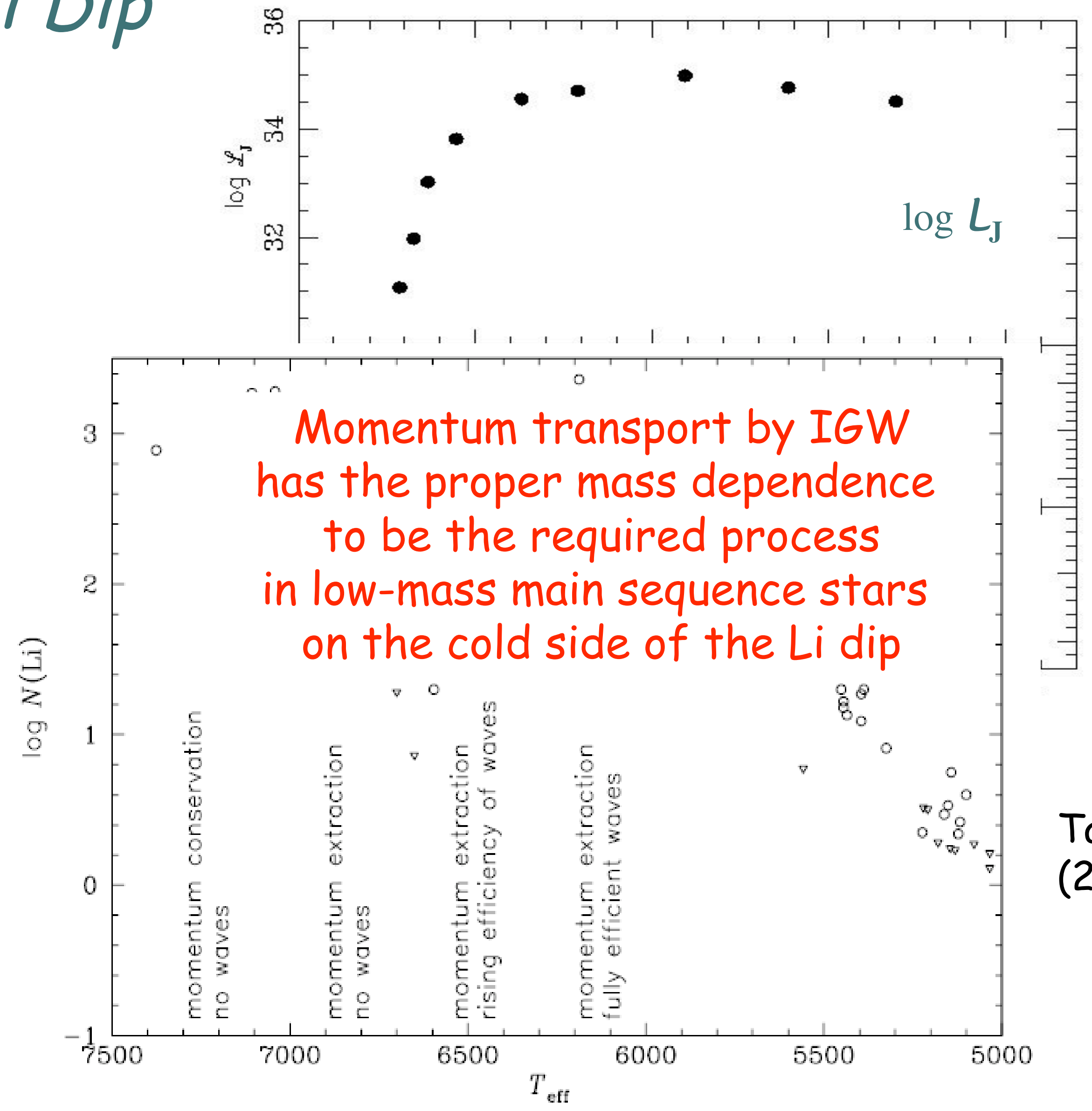


Talon & Charbonnel
(1998)



Internal gravity waves

The Li Dip

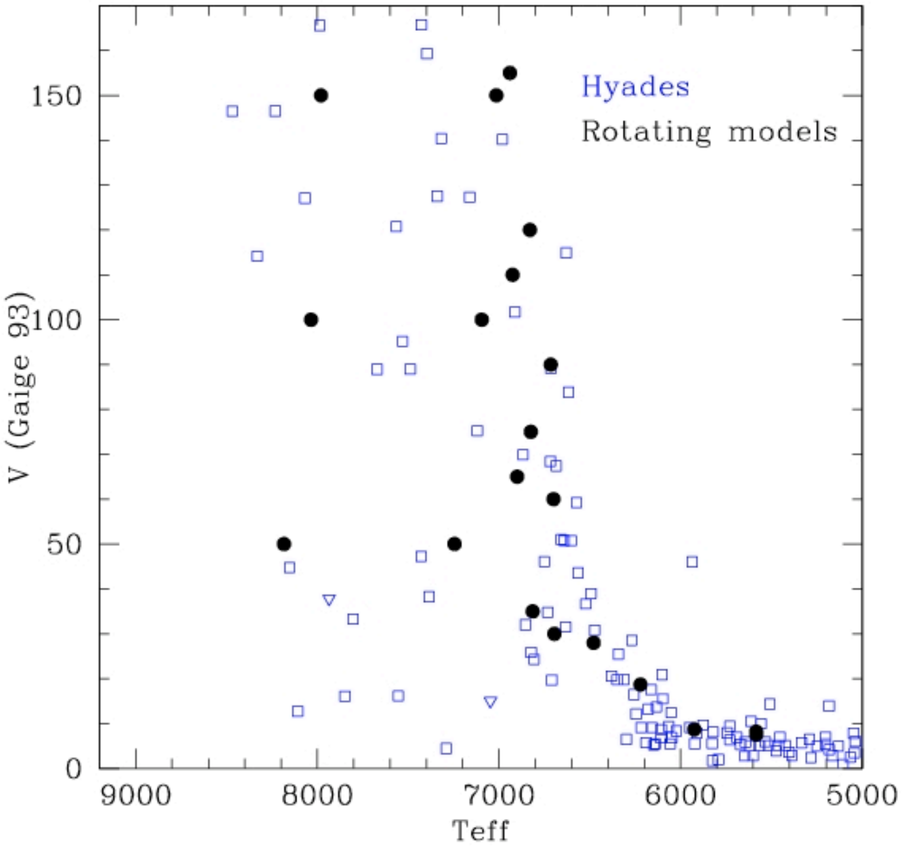


Momentum transport by IGW
has the proper mass dependence
to be the required process
in low-mass main sequence stars
on the cold side of the Li dip

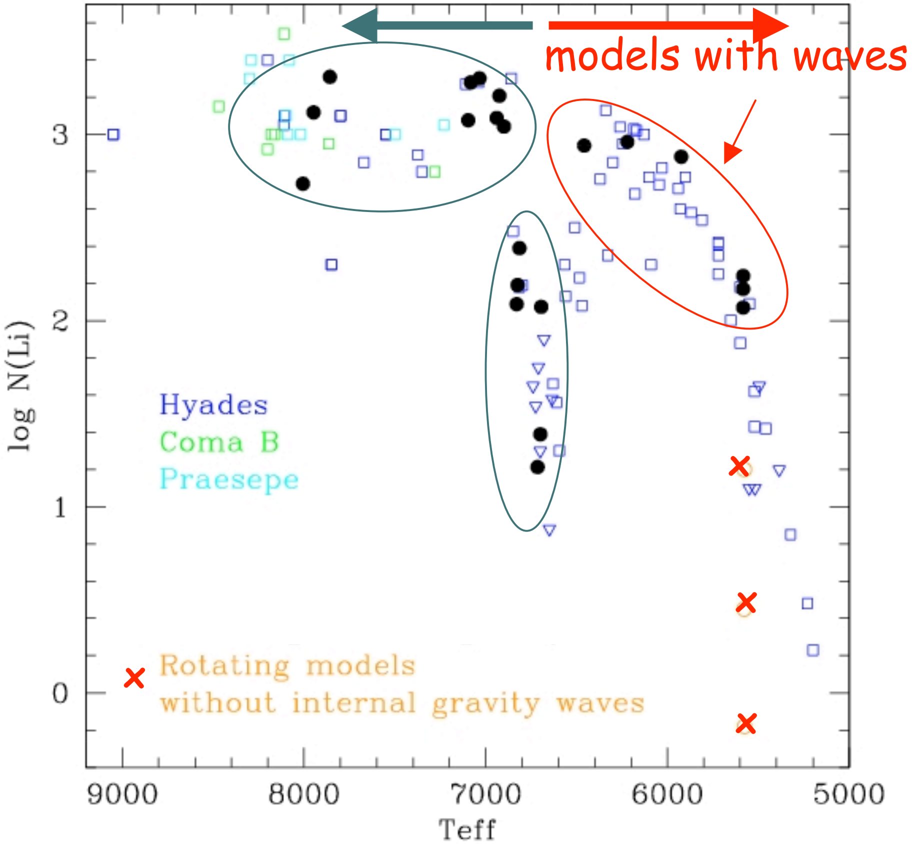
L_J: Net momentum luminosity at $0.03R_*$ below the surface convection zone as a function of $T_{\rm eff}$ (zams) for Pop I stars

Talon & Charbonnel (2003, 2004)

Internal gravity waves
The Li Dip



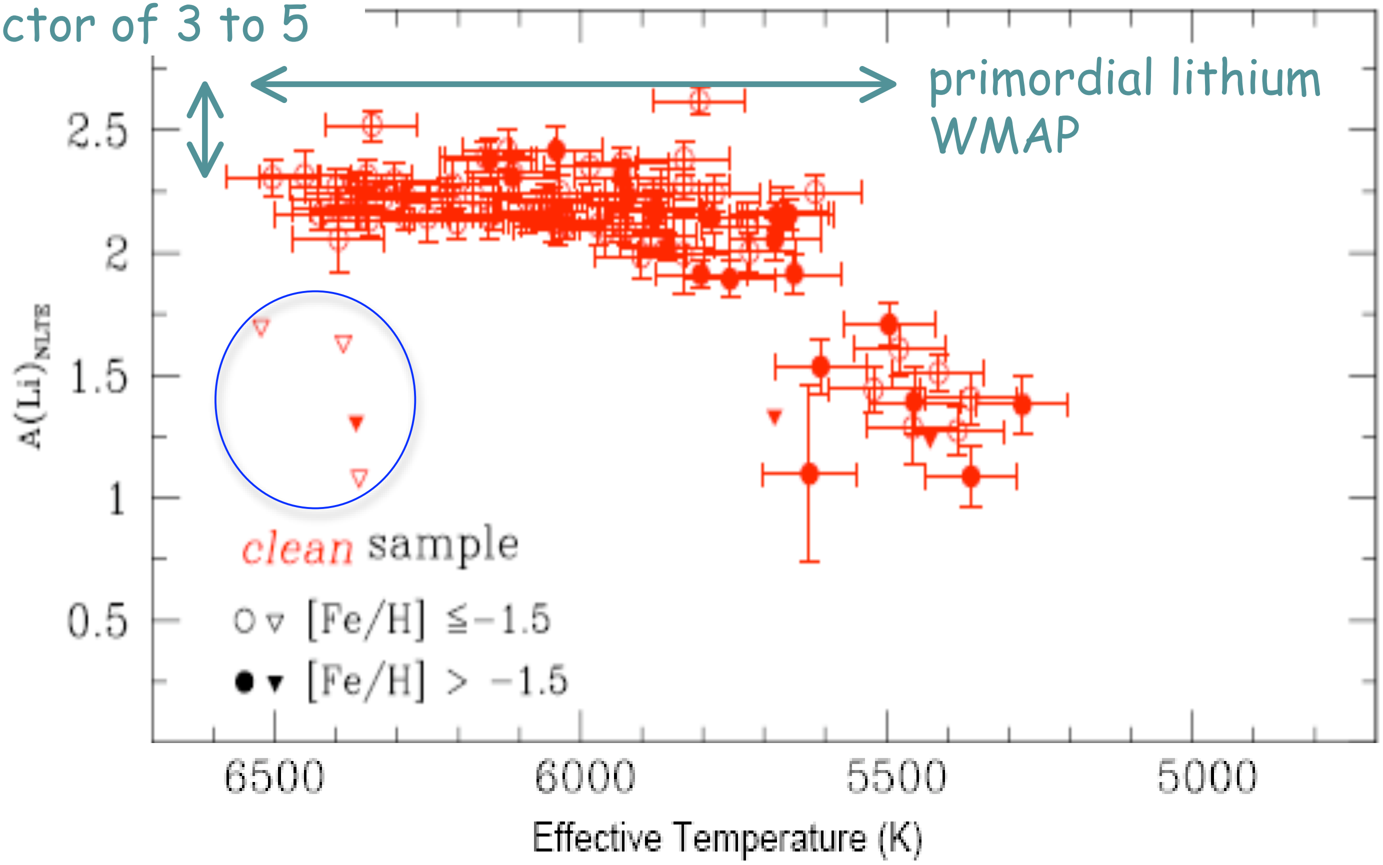
Charbonnel & Talon (2006)



Talon& Charbonnel (1998), Palacios, Talon, Charbonnel & Forestini (2003)

Internal gravity waves
Pop II stars

destruction by a
factor of 3 to 5

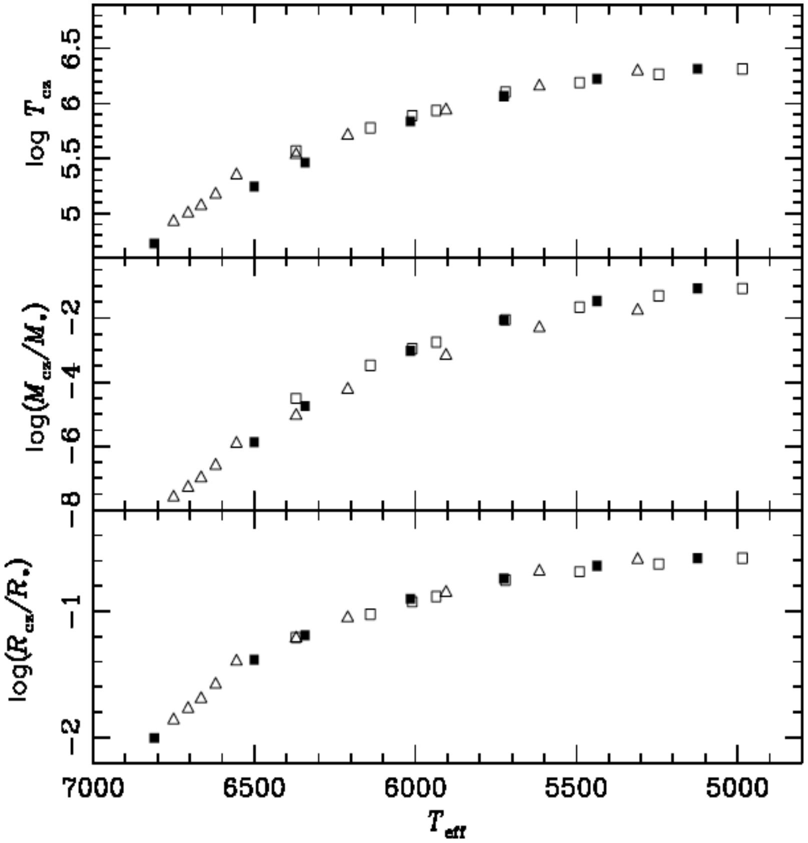


primordial lithium
WMAP

Charbonnel
& Primas
(2005)

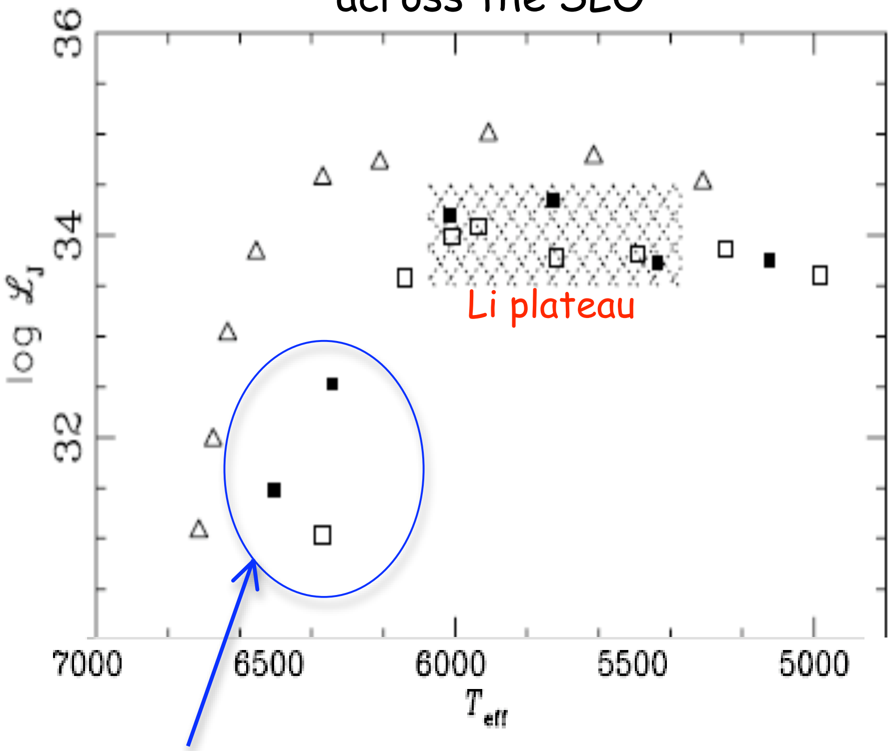
slightly more massive
stars, from « the
middle of the dip »

Internal gravity waves
Pop II stars



Open squares : Pop II stars on the zams
Black squares : Pop II stars at 10 Gyr
Open triangles : Pop I stars on the zams

net angular momentum luminosity
across the SLO



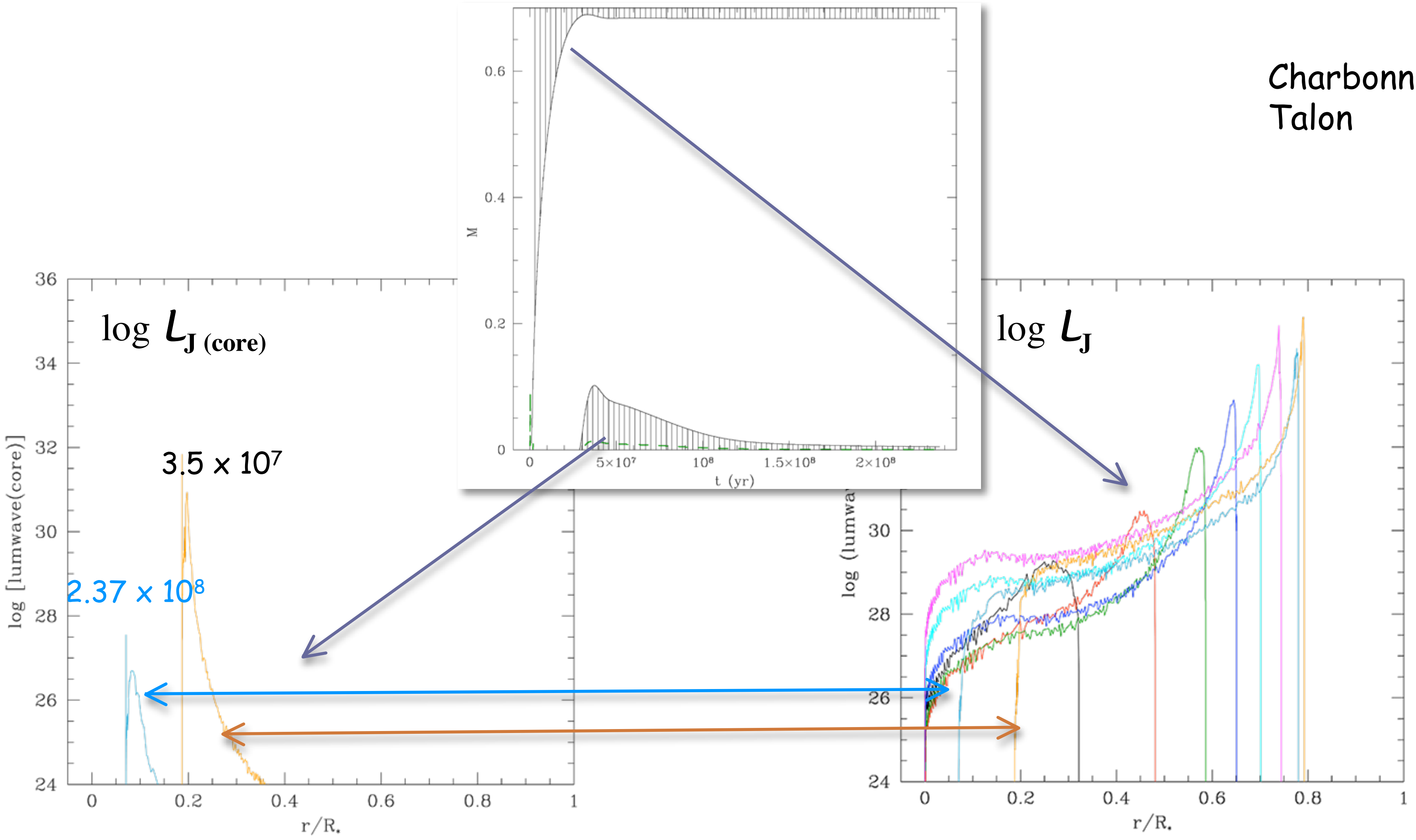
Li plateau

slightly more massive stars,
from « the middle of the
dip »

Talon & Charbonnel
(2004)

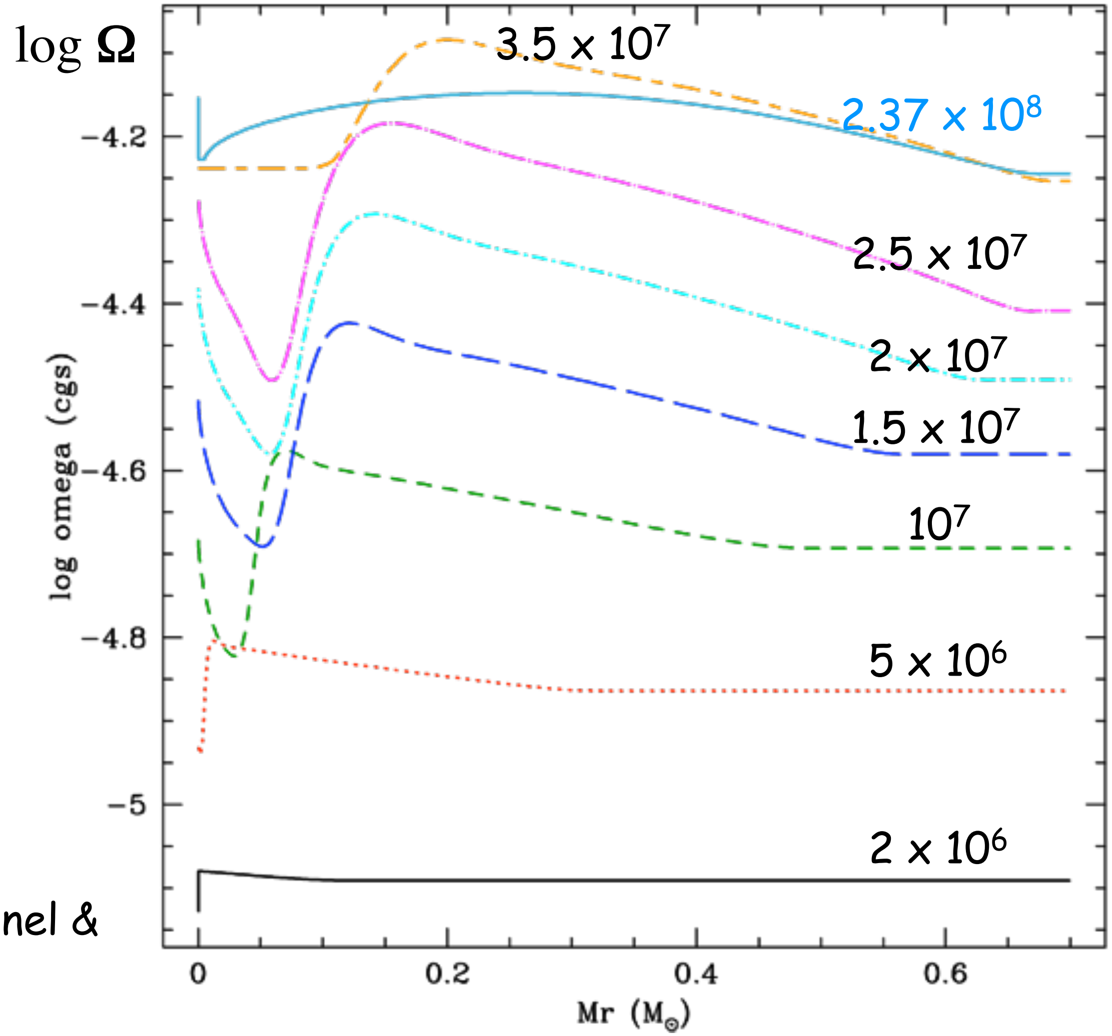
Internal gravity waves
A Pop II model - PMS evolution

Charbonnel & Talon



Internal gravity waves
A Pop II model – PMS evolution

PMS: Li destruction of 0.02 dex



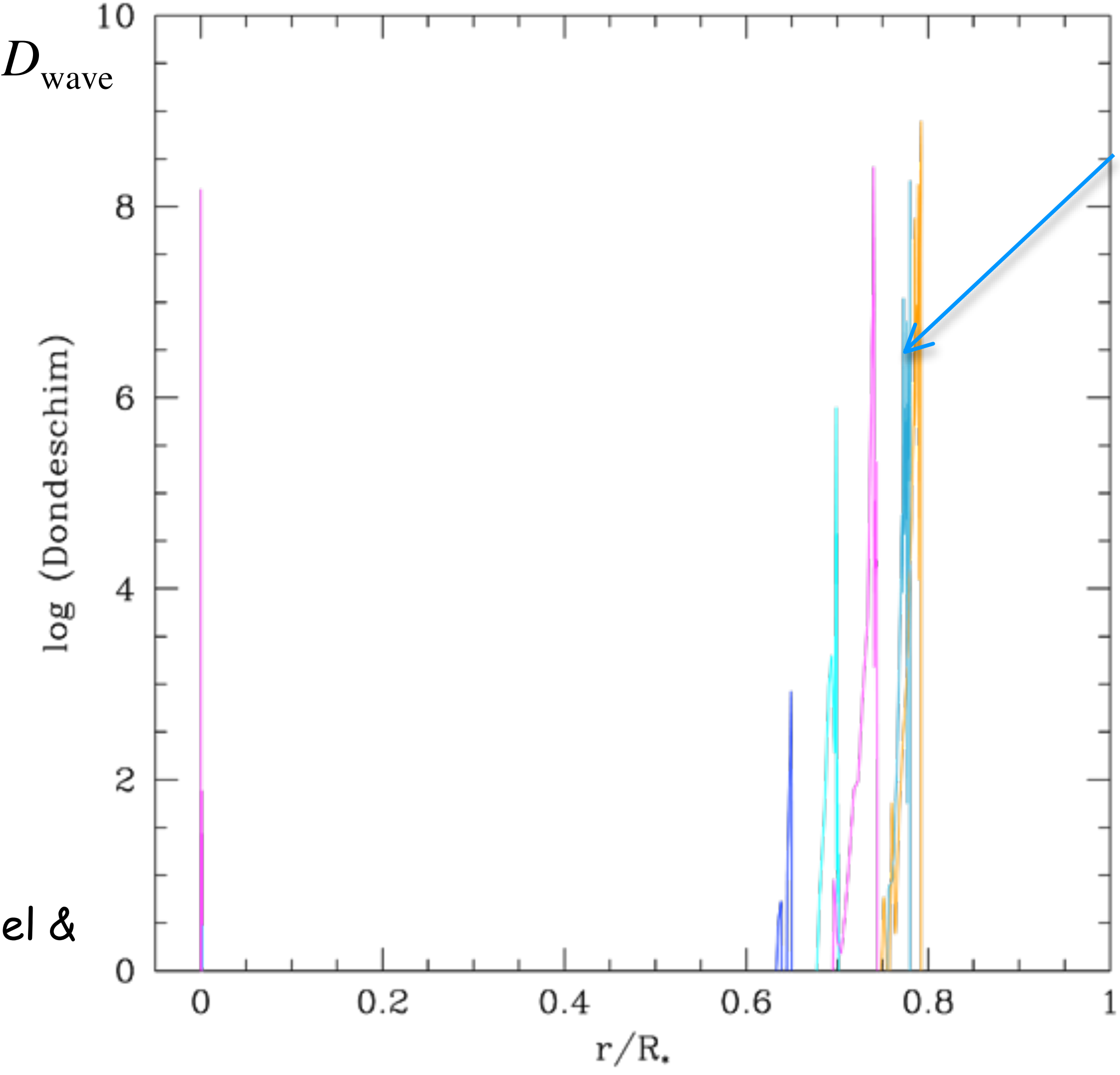
ZAMS
$v_{\rm surf} = 25$ km/s

disk lifetime of 2 Myr

Charbonnel & Talon

Internal gravity waves
A Pop II model - PMS evolution

$\log D_{\mathrm{wave}}$



ZAMS

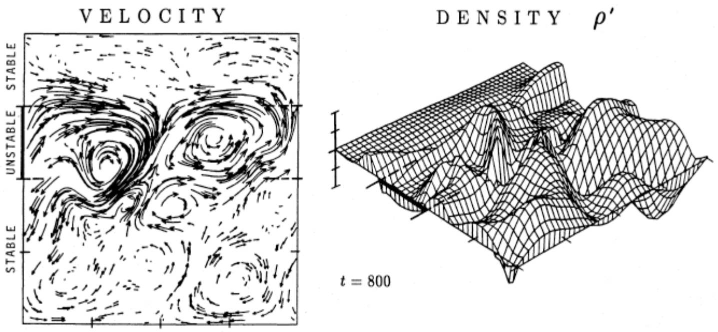
PMS: Li
destruction
of 0.02 dex

Charbonnel &
Talon

Open problems

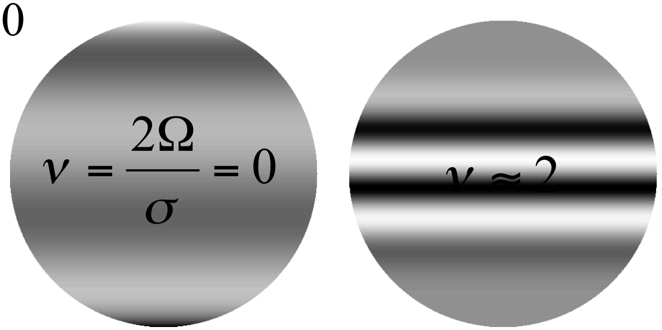
- Excitation

Hurlburt, Toomre, Massaguer (1986)



- Role of “fast” rotation

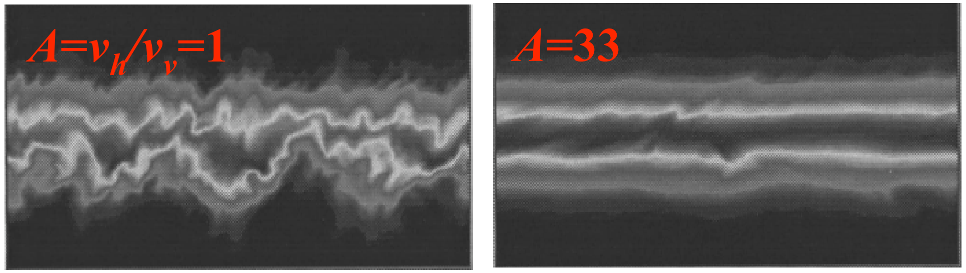
$l=5$, $m=0$



Pantillon, Talon, & Charbonnel (2007)

- Role of magnetic field
- Role of mixing by waves

- Reduction of vertical turbulent transport by anisotropic turbulence



Vincent, Michaud & Meneguzzi (1996)

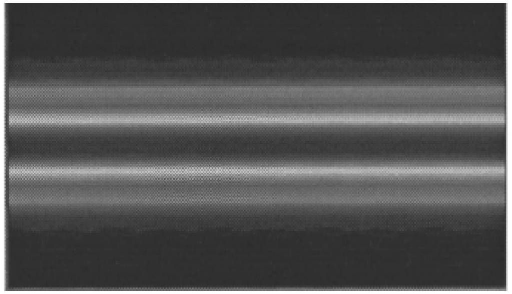
- Role of the Rossby radius

Hurricane Felix over the Caribbean Sea

- Also simply need complete evolutions with current physics!

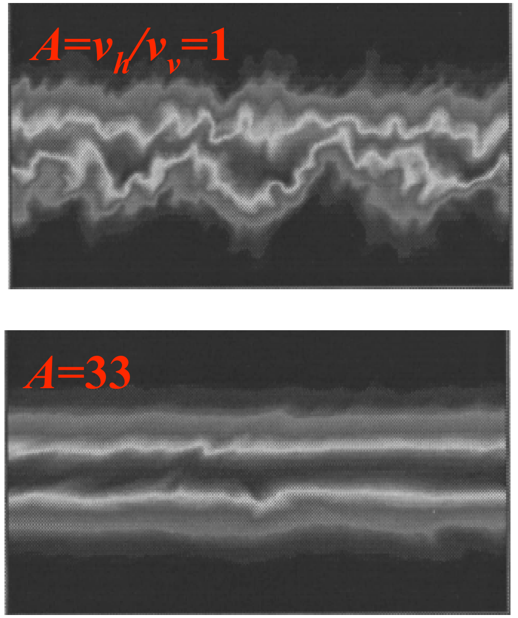
Open problems

- Reduction of vertical turbulent transport by anisotropic turbulence



pure atomic diffusion

Vincent, Michaud & Meneguzzi (1996)



atomic diffusion + turbulence

- In Am stars, required to reduce rotational mixing (Talon, Richard, Michaud 2005) ;
- In massive stars, would reduce mixing too much;
- Would help to get a Li plateau in Sun-like stars and no Be destruction.

Open problems

- Reduction of vertical turbulent transport by anisotropic turbulence
- Role of the Rossby radius (length scale at which rotational effects become as important as buoyancy effects in the evolution of the flow about some disturbance)

$$L_{\text{Rossby}} = \frac{NH_P}{f_0}$$

for the Sun (or slow rotator):

$$L_{\text{Rossby}} \approx \text{R}_{\odot}$$

for a massive star:

$$L_{\text{Rossby}} << \text{R}_{*}$$

Hurricane Felix over the Caribbean Sea